Conformal Carroll scalars with boosts

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Based on
[SB, G. Oling, W. Sybesma, B. Søgaard, arXiv: 2207.03468]

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Motivations

Find explicit actions for conformal Carroll theories

- Conformal Carroll isomorphic to BMS
- Few examples of flat space candidate dual field theories
- Include local Carroll boost symmetry

Use a small c expansion to build the Carroll equivalent of

$$S = -\frac{1}{2} \int d^d x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{(d-2)}{4(d-1)} R \phi^2 \right] \quad (1)$$

[Gupta, Suryanarayana, 2020][Bagchi, Banerjee, Dutta, Kolekar, Sharma, 2022][Rivera-Betancour, Vilatte, 2022][Saha, 2022]

Outline

Carroll symmetry

Lorentz boosts

$$t \to t + \beta x \,, \qquad x \to x + \beta t \tag{2}$$

Carroll boosts ($c \to 0$ limit) [Levi-Leblond, 1965][Sen Gupta, 1966]

$$t \to t + \lambda x \,, \qquad x \to x \tag{3}$$

Boost Ward identity implies [Henneaux, Salgado-Rebodello, 2021][De Boer, Hartong, Obers, Sybesma, Vandoren, 2021]

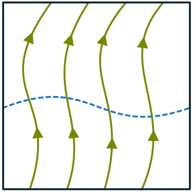
$$\langle \phi(t,x)\phi(0,0)\rangle = \begin{cases} f(t)\delta(x) & \text{timelike} \\ g(x) & \text{spacelike} \end{cases} \tag{4}$$

Carroll geometry

Geometry defined by a timelike vector field v^μ and a spatial metric $h_{\mu\nu}$ [Duval, Gibbons, Horvathy, Zhang, 2014][Hartong, 2015][Ciambelli, Marteau, Petkou, Petroupoulos, Siampos, 2018][Hansen, Obers, Oling, Søgaard, 2021]

Complemented with τ_μ and $h^{\mu\nu}$ transforming under local boosts

$$v^\mu h_{\mu\nu} = 0, \quad \tau_\mu h^{\mu\nu} = 0, \qquad v^\mu \tau_\mu = -1, \quad h^{\mu\rho} h_{\rho\nu} = \delta^\mu_\nu + v^\mu \tau_\nu \tag{5}$$



Extrinsic curvature and acceleration

$$K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_v h_{\mu\nu}\,, \qquad a_\mu = 2 v^\rho \partial_{[\mu}\tau_{\rho]} \tag{6}$$

Connection satisfying $\tilde{\nabla}_\mu v^\nu = 0$ and $\tilde{\nabla}_\rho h_{\mu\nu} = 0$
[Bekaert, Morand, 2015][Hartong, 2015]

$$\tilde{\Gamma}^\rho_{\mu\nu} = -v^\rho \partial_{(\mu}\tau_{\nu)} - v^\rho \tau_{(\mu}\mathcal{L}_v \tau_{\nu)} + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu}\right) - h^{\rho\sigma}\tau_\nu K_{\mu\sigma}$$

Energy-momentum tensor

$$T^v_\mu \equiv -\frac{1}{e}\frac{\delta S}{\delta v^\mu}\,, \qquad T^h_{\mu\nu} \equiv -\frac{2}{e}\frac{\delta S}{\delta h^{\mu\nu}}\,, \qquad e = \det(\tau_\mu, h_{\mu\nu}) \tag{7}$$

Boost-invariant combination [Hartong, Kiritsis, Obers, 2014]

$$T^\mu{}_\nu = -v^\mu T^v_\nu - h^{\mu\rho} T^h_{\rho\nu} \tag{8}$$

Ward identities

- Weyl

$$T^\mu_\mu = 0 \tag{9}$$

- Boost (in adapted coordinates (t, x^i))

$$T^i{}_0 = 0 \tag{10}$$

Pre-ultra local parametrization of the geometry

Get Carroll geometry from $c \to 0$ expansion of Lorentzian geometry (method based on [Hansen, Hartong, Obers, 2020][Hansen, Obers, Oling, Søgaard, 2021])

$$g_{\mu\nu} = -c^2 T_\mu T_\nu + \Pi_{\mu\nu} \,, \qquad g^{\mu\nu} = -\frac{1}{c^2} V^\mu V^\nu + \Pi^{\mu\nu} \tag{11}$$

Carroll geometry appears at leading order in the expansion

$$\begin{aligned} V^\mu &= v^\mu + \mathcal{O}(c^2), \qquad & \Pi_{\mu\nu} &= h_{\mu\nu} + \mathcal{O}(c^2) \\ T_\mu &= \tau_\mu + \mathcal{O}(c^2) \,, \qquad & \Pi^{\mu\nu} &= h^{\mu\nu} + \mathcal{O}(c^2) \end{aligned} \tag{12}$$

Carroll connection from expansion of Levi-Civita connection compatible with $g_{\mu\nu}$

Conformal Carroll actions

Pre-ultra local parametrization of the action

Relativistic scalar with conformal coupling

$$S = -\frac{1}{2}\int d^d x\,\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{(d-2)}{4(d-1)}R\phi^2\right] \tag{13}$$

Perform the pre-ultra local parametrization

$$\begin{aligned} S = -\frac{c}{2}\int d^d x\,E\Bigg[&\left(-\frac{1}{c^2}V^\mu V^\nu + \Pi^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi \\ &+\frac{(d-2)}{4(d-1)}\left(\frac{1}{c^2}\left[\mathcal{K}^{\mu\nu}\mathcal{K}_{\mu\nu}+\mathcal{K}^2-2V^\mu\partial_\mu\mathcal{K}\right]\right. \\ &\left.+\Pi^{\mu\nu}\tilde{R}_{\mu\nu}-\tilde{\nabla}_\mu(V^\nu\Pi^{\mu\rho}T_{\nu\rho})+\frac{c^2}{4}\Pi^{\mu\nu}\Pi^{\rho\sigma}T_{\mu\rho}T_{\nu\sigma}\right)\phi^2\Bigg] \end{aligned} \tag{14}$$

No expansion yet!

Timelike action

In the Carroll limit $c \to 0$, leading order terms give timelike action

$$S_t = -\frac{1}{2} \int d^d x \, e \left[-(v^\mu \partial_\mu \phi)^2 + \frac{(d-2)}{4(d-1)} \left(K^{\mu\nu} K_{\mu\nu} + K^2 - 2 v^\rho \partial_\rho K \right) \phi^2 \right] \quad (15)$$

Flat space propagator $\sim |t| \, \delta(\vec{x})$ of timelike form

Symmetries:

- Diffeomorphisms
- Weyl transformations
- Local Carroll boosts

Energy-momentum tensor satisfies

$$T^i_{\ 0} = 0 \,, \qquad T^\mu_{\ \mu} = 0 \quad (16)$$

Spacelike action

Alternative $c \to 0$ limit using Lagrange multipliers gives spacelike action

$$S_s = -\frac{1}{2}\int d^d x e \left[h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{(d-2)}{4(d-1)}\left(h^{\mu\nu}\tilde{R}_{\mu\nu} - \tilde{\nabla}_\mu a^\mu\right)\phi^2 \right. \\ \left. + \chi\left(v^\mu\partial_\mu\phi - \frac{(d-2)}{2(d-1)}K\phi\right) + \chi^{\mu\nu}K_{\mu\nu}\phi \right] \tag{17}$$

Constraints ⇒ Extrinsic curvature must be pure trace

$$K_{\mu\nu} = \frac{h_{\mu\nu}}{d-1}K \tag{18}$$

Timelike action vanishes on the constraint surface
⇒ The two actions cannot coexist at the same time!
(As opposed to [Gupta, Suryanarayana, 2020] [Rivera-Betancour, Vilatte, 2022])

Properties of the spacelike action

Symmetries (crucial to use constraints!):

- Diffeomorphisms
- Weyl transformations
- Local Carroll boosts

Energy-momentum tensor satisfies

$$T^{i}_{\ 0} = 0\,, \qquad T^{\mu}_{\ \mu} = 0 \tag{19}$$

Dimensional reduction

Integrate out the constraints

$$S_s = -\frac{1}{2} \int d^{d-1}x \, \sqrt{h} \left[h^{ij} \partial_i \hat{\phi} \partial_j \hat{\phi} + \frac{(d-3)}{4(d-2)} \hat{R} \hat{\phi}^2 + \frac{1}{4(d-1)(d-2)} A^{-2} \hat{R}_{A^{-2} h_{ij}} \hat{\phi}^2 \right]$$

$$A \equiv \int_v d\tau \,, \qquad \hat{\phi} \equiv A^{1/2} \phi \tag{20}$$

$\hat{R}$ Ricci scalar of Levi-Civita connection compatible with $h_{\mu\nu}$

Euclidean Weyl symmetry manifest!

Conclusions

- Timelike and spacelike conformal Carroll scalar actions
- Well-defined stress tensor satisfying

$$T^i_{\ 0} = 0 \,, \qquad T^\mu_{\ \mu} = 0$$

Future developments

- Build conformal Carroll $\Leftrightarrow$ CCFT dictionary [Donnay, Fiorucci, Herfray, Ruzziconi, 2022][Bagchi, Banerjee, Basu, Dutta, 2022]
- Explicit computation of Weyl anomalies in scalar theories
- General classification of Weyl anomalies

Timelike anomalies (parity-even):
[SB, Oling, Sybesma, Søgaard, in progress][Arav, Chapman, Oz, 2014]

$$\langle T^{\mu}_{\ \mu} \rangle = \begin{cases} \emptyset & d=2 \\ \emptyset & d=3 \\ b_1\left(-7\mathrm{Tr}(K^4)+\frac{1}{3}K\mathrm{Tr}(K^3)+\mathrm{Tr}(K^2)(\mathcal{L}_v K)+(\mathcal{L}_v K)^2\right)+b_2(\dots) & d=4 \\ \dots \end{cases}$$

Thank you!