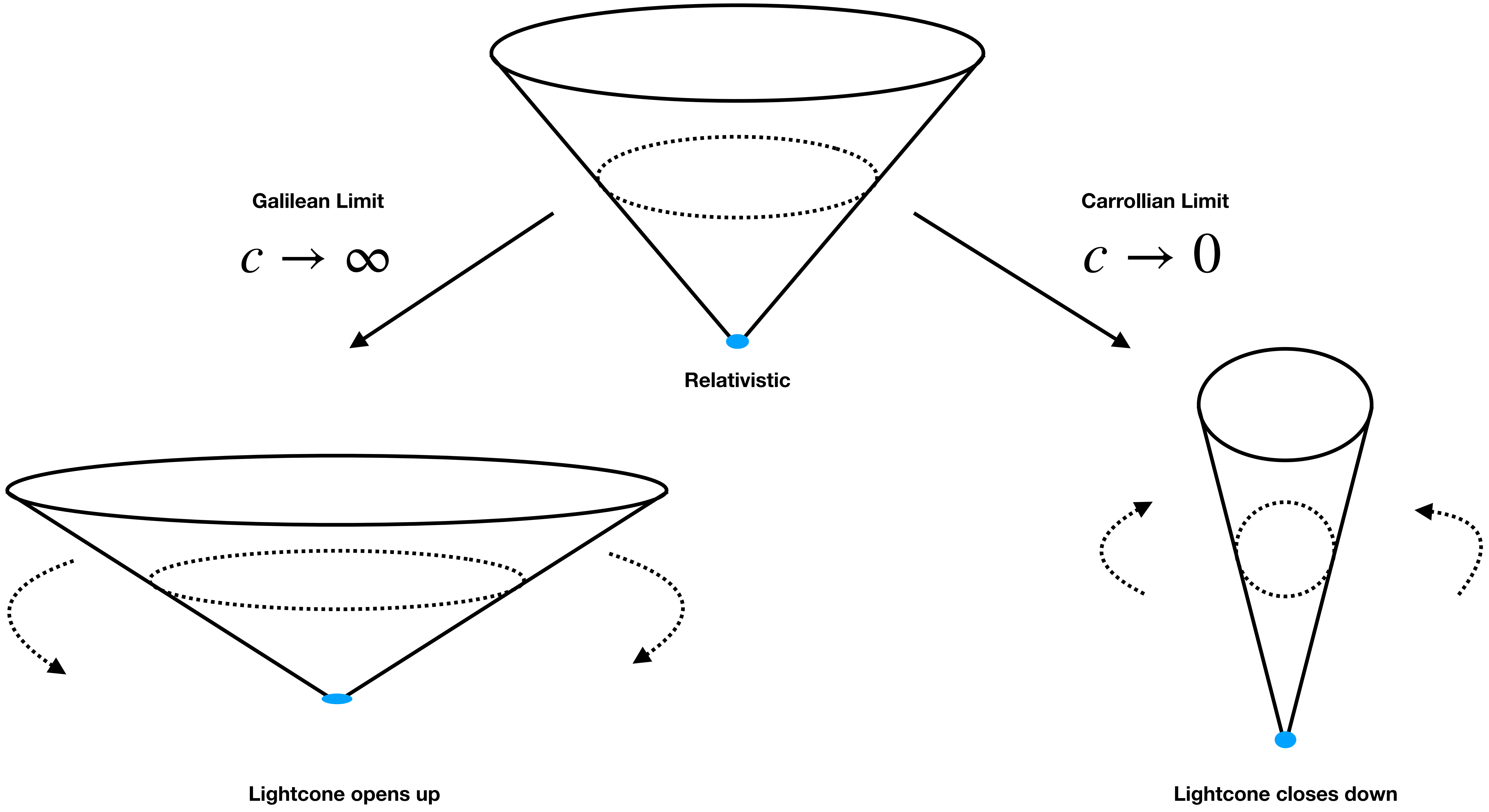
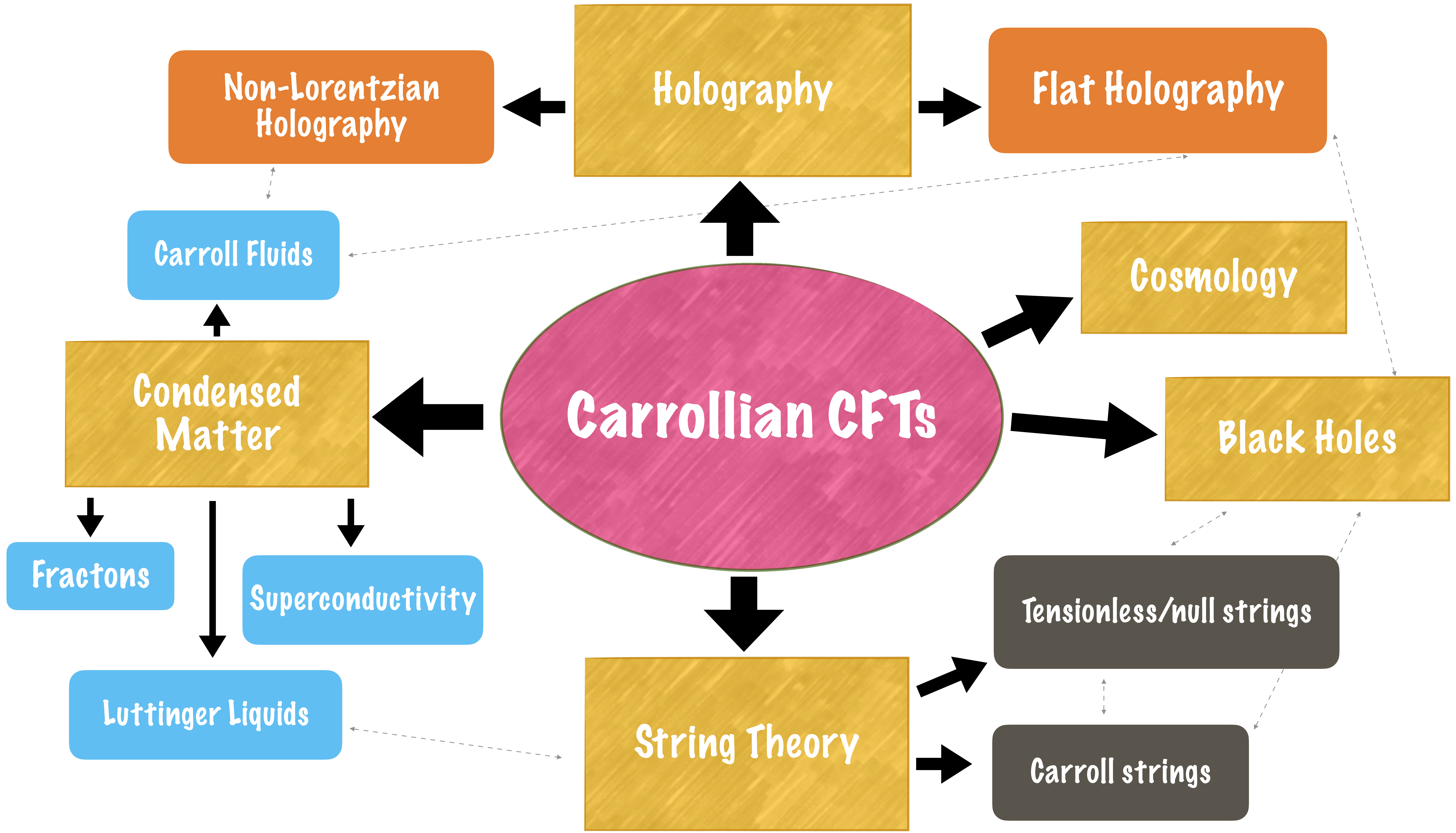
Unbearable Effectiveness of Carroll CFTs

Arjun Bagchi
IIT Kanpur

Non Lorentzian Limits



* We are familiar with Galilean limits.
* Here we would be interested in the diametrically opposite one, the Carroll limit.



Non-Lorentzian Holography
Holography
Flat Holography
Carroll Fluids
Cosmology
Condensed Matter
Carrollian CFTs
Black Holes
Fractons
Superconductivity
Tensionless/null strings
Luttinger Liquids
String Theory
Carroll strings

Today ...

We will give a brief overview of a few of these:

* Flat holography
* Tensionless strings
* Carroll Fermions

Flat Holography

Carroll and Conformal Carroll Symmetry: The algebraic way

* Carroll algebra: Inonu-Wigner contraction of Poincare algebra when $c \to 0$
* This can be achieved by $x^i \to x^i, \quad t \to \epsilon t, \quad \epsilon \to 0$
* Carroll generators: $H = \partial_t, \quad P_i = \partial_i, \quad C_i = x_i\partial_t, \quad J_{ij} = x_i\partial_j - x_j\partial_i.$
* The algebra: $[J_{ij}, J_{kl}] = 4\delta_{[i[k}J_{l]j]}, [J_{ij}, P_k] = 2\delta_{k[j}P_{i]}, [J_{ij}, C_k] = 2\delta_{k[j}C_{i]}, [C_i, P_j] = -\delta_{ij}H.$
* Crucially:$[C_i, C_j] = 0$. Reflects non-Lorentzian nature of the algebra.
* Conformal extension: $D = t\partial_t + x_i\partial_i, \quad K_0 = x_i x_i\partial_t, \quad K_i = 2x_i(t\partial_t + x_j\partial_j) - x_j x_j\partial_i.$
* Conformal Carroll algebra: $[D, P_i] = -P_i, \ [D, H] = -H \ [D, K_i] = K_i, \ [D, K_0] = K_0,$
  $[K_0, P_i] = -2C_i \ [K_i, H] = -2C_i, \ [K_i, P_j] = -2\delta_{ij}D - 2J_{ij}.$
* Can be given an infinite dimensional lift in all dimensions.

Carroll & Conformal Carroll Symmetry: The geometric way

* Start with Minkowski spacetime: $ds^2 = -c^2dt^2 + (dx^i)^2$ and send speed of light to zero.
* Metric degenerates

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 \\ 0 & I_{d-1} \end{pmatrix} \to \tilde{h}_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & I_{d-1} \end{pmatrix}, \quad \eta^{\mu\nu} = \begin{pmatrix} -1/c^2 & 0 \\ 0 & I_{d-1} \end{pmatrix} \quad -c^2\eta^{\mu\nu} \to \Theta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0_{d-1} \end{pmatrix}$$

* Also: $\Theta^{\mu\nu} = \theta^\mu\theta^\nu \qquad \tilde{h}_{\mu\nu}\theta^\nu = 0.$ Henneaux 1979
* A Carroll manifold is defined by a quadruple $(\mathcal{C}, \tilde{h}, \theta, \nabla)$ Duval, Gibbons, Horvathy 2014
  - $\mathcal{C}$ is a d dimensional manifold, on which one can choose a coordinate chart (t, x^i).
  - $\tilde{h}$ is a covariant, symmetric, positive, tensor field of rank $d-1$ and of signature $(0, \underbrace{+1, \ldots, +1}_{d-1})$.
  - θ is a non-vanishing vector field which generates the kernel of $\tilde{h}$.
  - ∇ is a symmetric affine connection that parallel transports both $\tilde{h}_{\mu\nu}$ and θ^ν.
* Carroll Lie algebra: $\mathcal{L}_\xi \tilde{h}_{\mu\nu} = 0, \quad \mathcal{L}_\xi\theta = 0.$ Conformal Carroll Lie algebra: $\mathcal{L}_\xi\tilde{h} = \lambda\tilde{h}, \quad \mathcal{L}_\xi\theta = -\frac{\lambda}{2}\theta.$

Flat space and BMS symmetries

* Asymptotic symmetries of flat space at null infinity is given by the Bondi-Metzner-Sachs (BMS) group.

* In 3 and 4 dimensions, the BMS group is infinite dimensional.

* In 3 dimensions, the BMS_3 algebra reads:

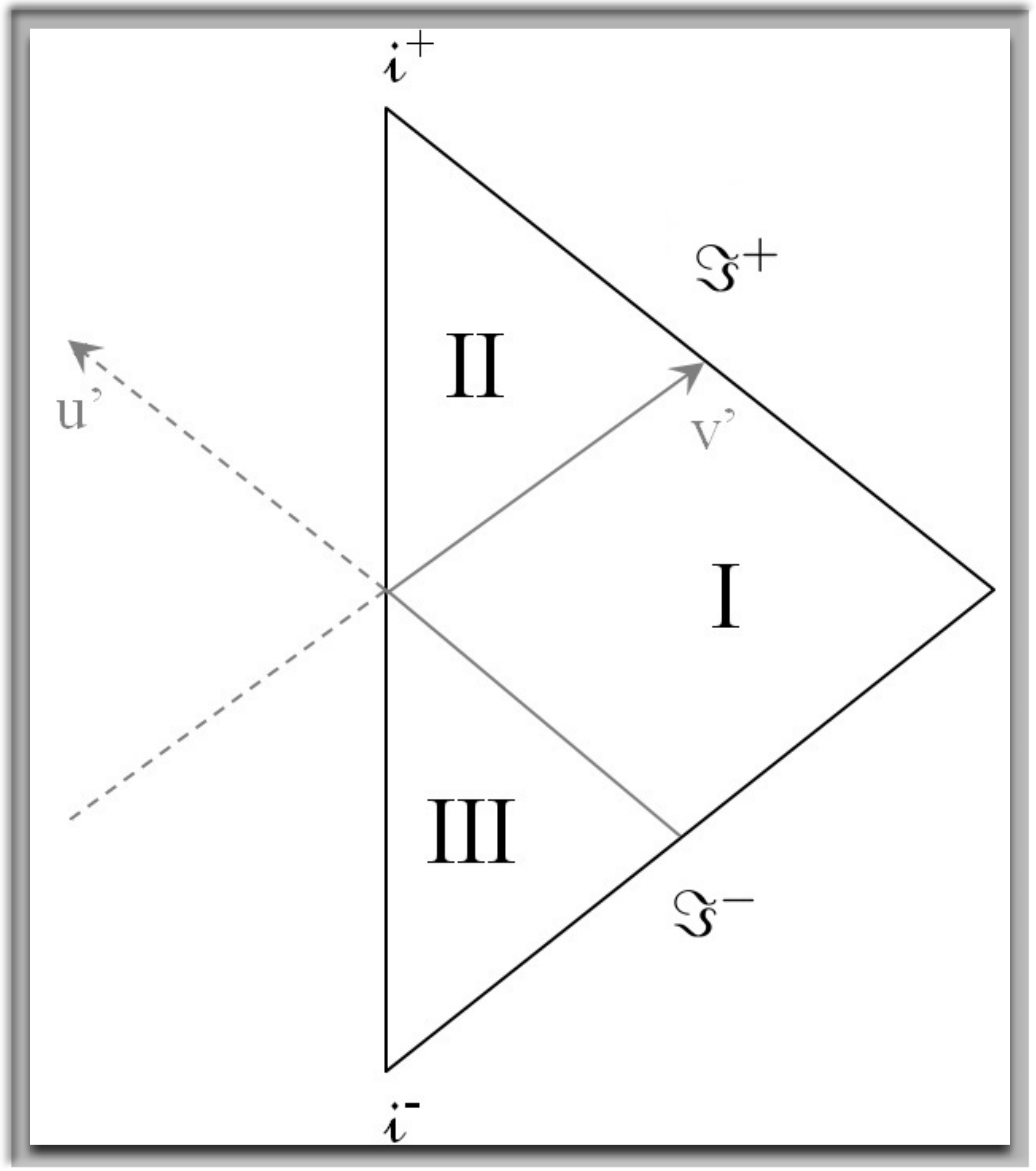
$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3 - n)$$
$$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3 - n)$$
$$[M_n, M_m] = 0.$$

* M's: supertranslations. Angle dependent translations along the null direction.

* L's: superrotations. Diffeos of the circle at infinity.

* For Einstein gravity, $c_L = 0, \quad c_M = \frac{3}{G}$

Barnich, Compere 2006



Penrose Diagram of Minkowski spacetime

Asymptotic Symmetries of 4d Flat Spacetime

* In 4d, the BMS_4 algebra is a bit more involved.

$$[L_n, L_m] = (n-m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m}$$
$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{n+r,s}, \quad [\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,n+s}$$
$$[M_{r,s}, M_{t,u}] = 0.$$

* Two Virasoros and supertranslations with two legs.
* Complications regarding central charges, which we will studiously avoid for now.

The Connection

AB 2010;
Duval, Gibbons, Horvathy 2014.

$$\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}.$$

Conformal Carroll algebra in d-dimensions is isomorphic to the BMS algebra in (d+1) dimensions

From AdS to Flatspace

* Can obtain flat space by taking the radius of AdS to infinity.
* Start with 2 copies of Virasoro algebra that form asymptotic symmetries of AdS3.

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\bar{\mathcal{L}}_n, \bar{\mathcal{L}}_m] = (n-m)\bar{\mathcal{L}}_{n+m} + \frac{\bar{c}}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\mathcal{L}_n, \bar{\mathcal{L}}_m] = 0$$

* The central terms of the left and right copies: $c = \bar{c} = \dfrac{3\ell}{2G}$
* We take the following limit: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$
* Easy to see that this contracts 2 copies of Virasoro algebra to BMS3 algebra.
* The central terms $c_L = c - \bar{c} = 0$ and $c_M = \epsilon(c + \bar{c}) = \dfrac{3}{G}$ *Barnich, Compere 2006*
* **Flatspace limit in bulk = Carroll limit on boundary.** *AB, Fareghbal 2012*

Carrollian road to Minkowskian holography

* Field theory dual to Minkowski spacetimes should inherit its asymptotic symmetries.
* For D-dim Minkowski spacetimes, the dual theory should be a (D-1)-dim field theory living on the null boundary of flatspace. It should be a **(D-1)-dimensional Carrollian CFT**.
* We would have two separate tools to study these field theories.
    * The intrinsic way: use only symmetries of BMS.
    * The limiting way: use the Carrollian limit from relativistic CFTs.
* We will be attempting to understand aspects of flatspace from a field theory on $\mathcal{I}_+$.

Carrollian Holography: some checks of proposal

* Asymptotic density of states from field theory and bulk [AB, Detournay, Fareghbal, Simon 2012; Barnich 2012; AB, Basu 2013.]

* Multipoint correlation functions of EM tensor in boundary and bulk.
  * Novel phase transitions from zero-point functions. [AB, Detournay, Grumiller, Simon'13].
  * Matching of higher point correlations [AB, Grumiller, Merbis '15].

* Construction and matching of Entanglement Entropy [AB, Basu, Grumiller, Riegler '14; Jiang, Song, Wen '17; Hijano-Rabideau '17].

* Holographic Reconstruction of 3d flatspace [Hartong '15].

* Construction of bulk-boundary dictionary, matching of correlation functions of primary operators [Hijano-Rabideau '17; Hijano '18]

* BMS Characters & matching with 1-loop partition function [Oblak '15; Barnich, Gonzalez, Oblak, Maloney '15; AB, Saha, Zodinmawia '19]

* Asymptotic Structure constants from boundary and bulk [AB, Nandi, Saha, Zodinmawia '20]

* Generalisations
  * Flat Space Chiral Gravity: CS Gravity dual to chiral half of CFT. [AB, Detournay, Grumiller '12]
  * Higher spin theories in flat space. [Afshar, AB, Fareghbal, Grumiller, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13]

* Fluid-Gravity correspondence for flat space [Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18].

Ancient History

AB, Detournay, Fareghbal, Simon 2012.

See also Barnich 2012.

S=Area/4G for Flat Holography?

* Important early checks of AdS/CFT: CFT reproduces Black Hole entropy.
* Entropy of BTZ black holes = Entropy from Cardy formula in CFT2.
* Can we do something similar for holography in flat spacetimes?
* Yes! AB, Detournay, Fareghbal, Simon 2012. (See also Barnich 2012)
* We will quickly review this old work to remind people of one of the early successes of this programme.

BTZ Black holes and 2d CFT

* The non-extremal BTZ black hole is given by

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi + \frac{r_+ r_-}{\ell r^2}dt\right)^2$$

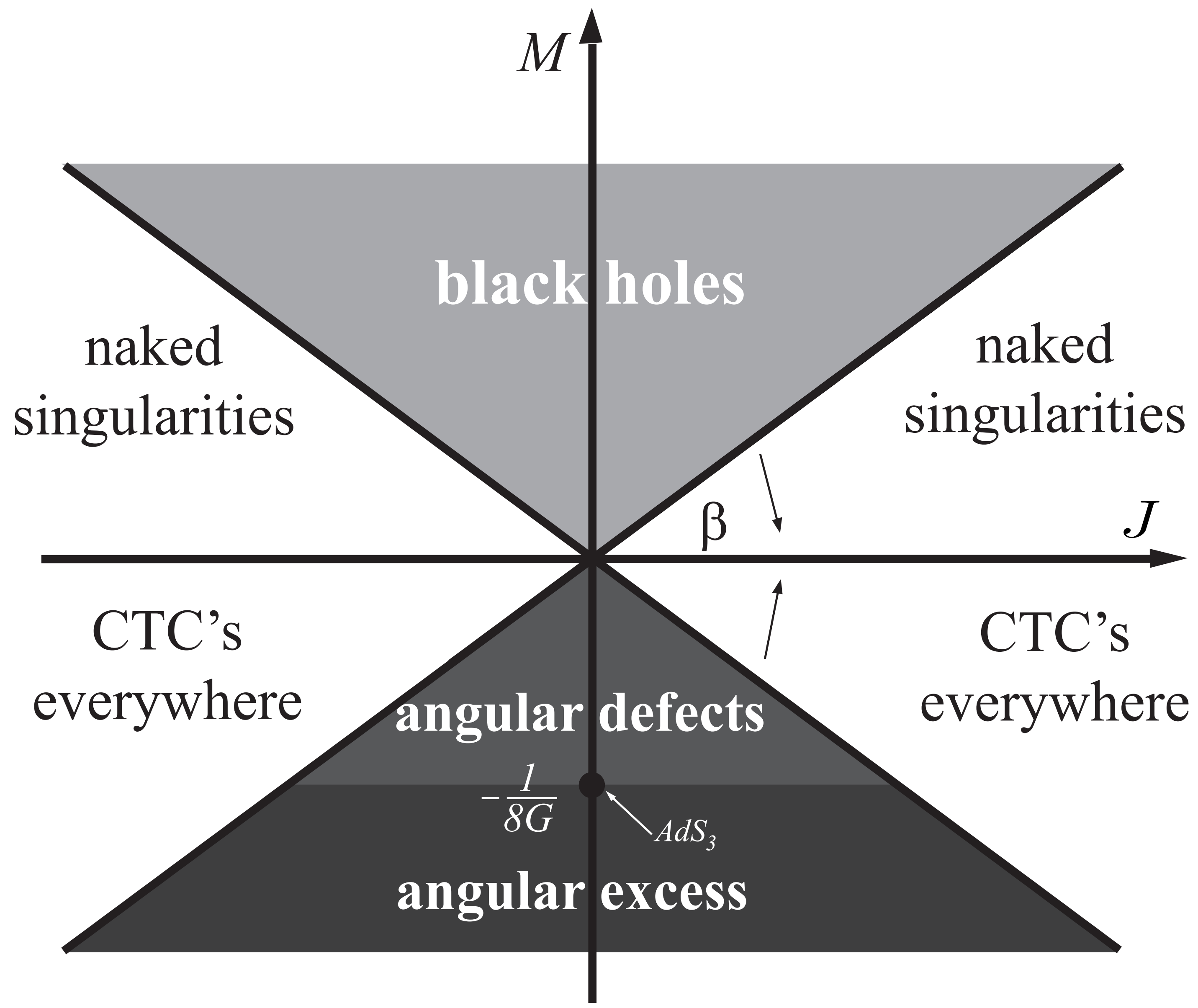
$$r_\pm = \sqrt{2G\ell(\ell M + J)} \pm \sqrt{2G\ell(\ell M - J)};$$

* Bekenstein-Hawking entropy: $S_{BH} = \frac{\text{Area of Horizon}}{4G} = \frac{\pi r_+}{2G}.$

* Cardy formula for 2d CFTs: $S_{CFT} = 2\pi\left(\sqrt{\frac{c\,h}{6}} + \sqrt{\frac{\bar{c}\,\bar{h}}{6}}\right).$

* Central terms for AdS3 and weights: $c = \bar{c} = \frac{3\ell}{2G}.\quad h = \frac{1}{2}(\ell M + J) + \frac{c}{24},\quad \bar{h} = \frac{1}{2}(\ell M - J) + \frac{\bar{c}}{24}$

* So ultimately: $S_{BH} = S_{CFT}$



Phase space of AdS3 solutions

Flat Space Cosmologies

* Take the radius of AdS to infinity. No Black holes in 3d flat spacetimes. What is happening?

* Outer horizon goes to infinity. Left with inside of BTZ black hole.

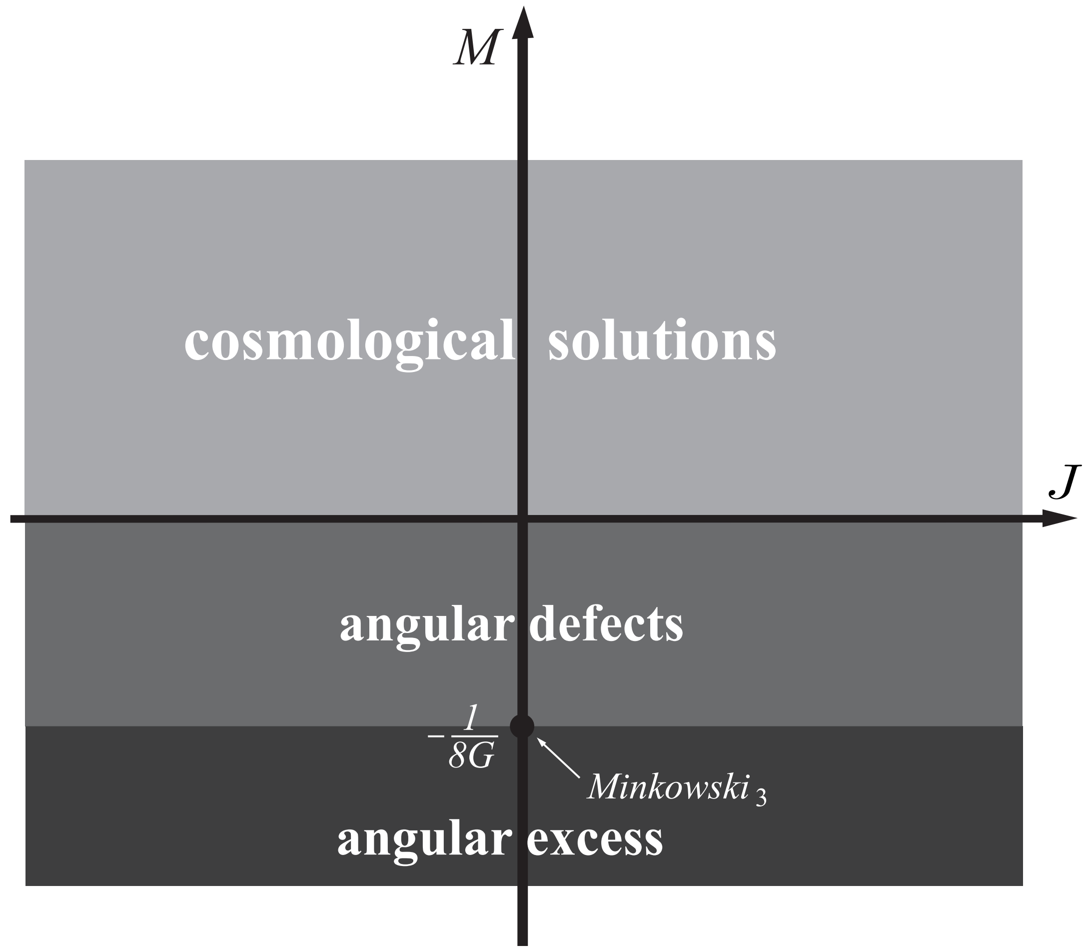
$$\ell \to \infty : r_+ \to \ell\sqrt{2GM} = \ell\hat{r}_+, \quad r_- \to r_0 = \sqrt{\frac{2G}{M}}\,J.$$

* Inner horizon survives. Cosmological solution with horizon. Flat Space Cosmology.

$$ds^2_{\text{FSC}} = \hat{r}_+^2 dt^2 - \frac{r^2\,dr^2}{\hat{r}_+^2(r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi$$

* Entropy:

$$\boxed{S_{\text{FSC}} = \frac{\text{Area of horizon}}{4G} = \frac{\pi r_0}{2G} = \frac{\pi J}{\sqrt{2GM}}}$$



Phase space of Min_3 solutions

BMS-Cardy formula and Entropy matching

* Label states of the 2d Carroll CFT: $L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle, \; M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$
* Partition function: $Z_{\text{CarrollCFT}} = \text{Tr}\exp\{2\pi i\,(\sigma L_0 + \rho M_0)\}$
* Carroll modular transformations: $\sigma \to \frac{a\sigma+b}{c\sigma+d}, \quad \rho \to \frac{\rho}{(c\sigma+d)^2}$
* Demand invariance of Z to derive BMS-Cardy formula

$$S^{(0)} = \ln d(\Delta,\xi) = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).$$

* Carroll Weights: $\xi = GM, \quad \Delta = J.$ Central Charges: $c_M = \frac{3}{G}, \, c_L = 0.$
* Putting things together: $S_{FSC} = S_{BMS-Cardy}$

Flat Holography : Aspects of dual theory

- Symmetry of 2d Carroll CFT:
$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3-n)$$
$$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3-n)$$
$$[M_n, M_m] = 0.$$
- Label states of the theory with $L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle,\ M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$
- We will build highest weight representations.
- BMS Primaries: $L_n|\Delta,\xi\rangle_p = M_n|\Delta,\xi\rangle_p = 0,\ \forall n > 0.$
- BMS modules are built out of these primary states by acting with raising operators.
- A general descent is of the form $L_{-1}^{k_1}L_{-2}^{k_2}....L_{-l}^{k_l}M_{-1}^{q_1}M_{-2}^{q_2}....M_{-r}^{q_r}|\Delta,\xi\rangle \equiv L_{\vec{k}}M_{\vec{q}}|\Delta,\xi\rangle$

Carroll CFT: Partition functions.

- Can define the theory on a cylinder. $L_n = ie^{in\phi}(\partial_\phi + in\tau\partial_\tau), \quad M_n = ie^{in\phi}\partial_\tau$
- The mapping from the plane to the cylinder: $x = e^{i\phi}, \quad t = i\tau e^{i\phi}$
- We can identify the end of the cylinder to define the theory on the torus.
- Partition function: $Z_{\text{CarrollCFT}} = \text{Tr}\exp\{2\pi i\,(\sigma L_0 + \rho M_0)\}$
- Look at Carroll limit of CFTs. 2d CFT partition function: $Z_{\text{CFT}} = \text{Tr}\; e^{2\pi i\zeta L_0}e^{-2\pi i\bar{\zeta}\bar{L}_0}$
- Relation between weights: $\Delta = h - \bar{h},\; \xi = \epsilon(h + \bar{h}).$
- In a convenient basis: $Z_{\text{CFT}} = \sum d_{\text{CFT}}(h,\bar{h})e^{2\pi i(\zeta h - \bar{\zeta}\bar{h})} = \sum d(\Delta,\xi)e^{2\pi i(\sigma\Delta - \frac{\rho}{\epsilon}\xi)}$
- Here $2\sigma = \zeta - \bar{\zeta}, \quad 2\rho = \zeta + \bar{\zeta}$
- We work with the assumption that $Z_{\text{CFT}} \to Z_{\text{CarrollCFT}}$ as $\epsilon \to 0$
- To keep the partition function finite, we need to scale $\rho \to \epsilon\rho$

Modular invariance in 2d Carroll CFTs

* BMS Partition function: $Z_{\text{BMS}} = \sum d(\Delta, \xi) e^{2\pi i(\sigma\Delta - \rho\xi)}$

* Any notion of BMS modular invariance? We again investigate the limit.

* Modular transformation in the original CFT: $\zeta \to \dfrac{a\zeta + b}{c\zeta + d}$ with $ad - bc = 1$

* In the BMS basis:

$$\sigma + \rho \to \frac{a(\sigma+\rho)+b}{c(\sigma+\rho)+d} = \frac{a\sigma+b}{c\sigma+d} + \frac{(ad-bc)\rho}{(c\sigma+d)^2} + \frac{(ad-bc)c\rho^2}{(c\sigma+d)^3} + \dots$$

* The contracted modular transformation reads:

$$\boxed{\sigma \to \frac{a\sigma+b}{c\sigma+d}, \quad \rho \to \frac{\rho}{(c\sigma+d)^2}}$$

* This is what we will call the **Carroll modular transformation**.

* Intrinsic interpretation=> S-transformation: Exchange of circles on the Euclidean torus.
[ala **Detournay-Hartman-Hofmann** for warped CFT. See e.g. **Song et al 2017**]

Invariance of Partition function

- Demand partition function is invariant under Carroll modular transformation and find consequences.

$$Z^0_{\text{BMS}}(\sigma,\rho) = \text{Tr}\; e^{2\pi i\sigma(L_0 - \frac{c_L}{2})} e^{2\pi i\rho(M_0 - \frac{c_M}{2})} = e^{\pi i(\sigma c_L + \rho c_M)} Z_{\text{BMS}}(\sigma,\rho)$$

- Carroll S-transformation: $(\sigma,\rho) \to \left(-\frac{1}{\sigma}, \frac{\rho}{\sigma^2}\right)$
- Invariance of the above quantity: $Z^0_{\text{BMS}}(\sigma,\rho) = Z^0_{\text{BMS}}\left(-\frac{1}{\sigma}, \frac{\rho}{\sigma^2}\right)$
- This translates to: $Z_{\text{BMS}}(\sigma,\rho) = e^{2\pi i\sigma\frac{c_L}{2}} e^{2\pi i\rho\frac{c_M}{2}} e^{-2\pi i(-\frac{1}{\sigma})\frac{c_L}{2}} e^{-2\pi i(\frac{\rho}{\sigma^2})\frac{c_M}{2}} Z_{\text{BMS}}\left(-\frac{1}{\sigma}, \frac{\rho}{\sigma^2}\right)$
- The density of states can be found with an inverse Laplace transformation

$$d(\Delta,\xi) = \int d\sigma d\rho\;\; e^{2\pi i\tilde{f}(\sigma,\rho)} Z\left(-\frac{1}{\sigma}, \frac{\rho}{\sigma^2}\right).$$

where $\tilde{f}(\sigma,\rho) = \frac{c_L\sigma}{2} + \frac{c_M\rho}{2} + \frac{c_L}{2\sigma} - \frac{c_M\rho}{2\sigma^2} - \Delta\sigma - \xi\rho.$

- In the limit of large charges, this integration can be done with a saddle point approximation.

BMS Cardy formula

- In the large charge limit, $\tilde{f}(\sigma,\rho) \to f(\sigma,\rho) = \frac{c_L}{2\sigma} - \frac{c_M \rho}{2\sigma^2} - \Delta\sigma - \xi\rho.$
- Value at the extremum is $f^{max}(\sigma,\rho) = -i\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).$
- BMS-Cardy formula is given by

$$\boxed{S^{(0)} = \ln d(\Delta,\xi) = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).}$$

Bagchi, Detournay, Fareghbal, Simon 2012.

- One can calculate leading logarithmic corrections to this.

$$S = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right) - \frac{3}{2}\log\left(\frac{\xi}{c_M^{1/3}}\right) + \text{constant} = S^{(0)} + S^{(1)}.$$

Bagchi, Basu 2013.

FSC entropy from dual theory

- The weights for the FSC: $\xi = GM + \frac{c_M}{24} \sim GM, \quad \Delta = J$
- Putting this back into the BMS-Cardy formula, we get $S_{\text{FSC}} = \frac{\pi J}{\sqrt{2GM}}$

Bagchi, Detournay, Fareghbal, Simon 2012; Barnich 2012

which is precisely what we obtained from the gravitational analysis.

- The log-correction is of the form $S^{(1)}_{\text{FSC}} = -\frac{3}{2}\log(2GM)$
- Total entropy: $$S_{\text{FSC}} = \frac{2\pi r_0}{4G} - \frac{3}{2}\log\left(\frac{2\pi r_0}{4G}\right) - \frac{3}{2}\log\kappa + \text{ constant}$$

Bagchi, Basu 2013.

Here $\kappa = \frac{\hat{r}^2}{r_0} = \frac{8GM}{r_0}$ is the surface gravity of FSC.

- Can also be obtained in the limit from the "inner" Cardy formula.

Riegler 2014; Fareghbal, Naseh 2014.

What's new? Bulk Scattering from Carroll CFTs

AB, Banerjee, Basu, Dutta 2022 (PRL)

* In asymptotically flat spaces, S-matrices are the observables of interest.
* Especially true in d>=4, where one has propagating DOF.
* Can we connect Carroll CFT correlations to S-matrix? YES!
* Interesting branches of correlators. "Weird" branch gives correct answer.
* We show this for d=3 boundary theory and d=4 bulk.
* Inspired by Pasterski-Shao map for Celestial CFTs. Use modified Mellin transformations.
* More details: See talk by SUDIPTA DUTTA tomorrow!
* Also talks by Laura, Romain, Adrien in this conference for another perspective.

3d Carrollian CFTs

Algebra on $\mathscr{I}^+$: $[L_n, L_m] = (n-m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m}$

$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{n+r,s}, \quad [\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,n+s} \qquad [M_{r,s}, M_{t,u}] = 0.$$

Representation (vector fields): $L_n = -z^{n+1}\partial_z - \frac{1}{2}(n+1)z^n u\partial_u \quad \bar{L}_n = -\bar{z}^{n+1}\partial_z - \frac{1}{2}(n+1)\bar{z}^n u\partial_u \quad M_{r,s} = z^r \bar{z}^s \partial_u$

Here z: stereographic coordinate on sphere, u: null direction.

Labelling of operators: $[L_0, \Phi(0)] = h\Phi(0), \quad [\bar{L}_0, \Phi(0)] = \bar{h}\Phi(0).$

Assume existence of Conformal Carroll primaries on $\mathscr{I}^+$

Highest weight representations: $[L_n, \Phi(0)] = 0, \quad [\bar{L}_n, \Phi(0)] = 0, \quad \forall n > 0, \quad [M_{r,s}, \Phi(0)] = 0, \quad \forall r, s > 0.$

Transformation rules for Carrollian primaries: $\delta_{L_n}\Phi_{h,\bar{h}}(u,z,\bar{z}) = \epsilon\left[z^{n+1}\partial_z + (n+1)z^n\left(h + \frac{1}{2}u\partial_u\right)\right]\Phi_{h,\bar{h}}(u,z,\bar{z})$

$$\delta_{M_{r,s}}\Phi_{h,\bar{h}}(u,z,\bar{z}) = \epsilon z^r \bar{z}^s \partial_u \Phi_{h,\bar{h}}(u,z,\bar{z}).$$

Scattering in 4d flatspace: Connections to 2d CFT

Consider massless particles. 4-momenta parametrised as:

$$p^\mu = \omega\left(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}\right), \; p^\mu p_\mu = 0$$

Mellin transformation: We also introduce a symbol ϵ which is equal to ± 1 if the particle is (outgoing) incoming.

$$\mathcal{M}\left(\{z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} S\left(\{\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i\}\right), \; \Delta \in \mathbb{C}, \; \sigma \in \frac{\mathbb{Z}}{2}$$

S is the S-matrix element for n massless particle scattering.

Also: $h = \dfrac{\Delta + \sigma}{2}, \; \bar{h} = \dfrac{\Delta - \sigma}{2}$

Using Lorentz transformation properties of the S-matrix, it can be shown that the LHS transforms like a correlation function of n primary operators of a 2d CFT.

[Pasterski-Shao(-Strominger), 2016]

4d Scattering: Modified Mellin Transformation

Under supertranslations: $u \to u' = u + f(z, \bar{z}), \;\; z \to z' = z, \;\; \bar{z} \to \bar{z}' = \bar{z}$

Under superrotations: $u \to u' = \left(\frac{dw}{dz}\right)^{\frac{1}{2}} \left(\frac{d\bar{w}}{d\bar{z}}\right)^{\frac{1}{2}} u, \;\; z \to z' = w(z), \;\; \bar{z} \to \bar{z}' = \bar{w}(\bar{z})$

Modified Mellin transformation:

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} e^{-i\epsilon_i \omega_i u_i} S\left(\{\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i\}\right), \;\; \Delta \in \mathbb{C}$$

[Banerjee 2017, Banerjee-Ghosh-Paul 2020]

Now defined in a 3d space with coordinates $(u, z, \bar{z})$. Transforms covariantly under BMS transformations

Used in Celestial holography since original Mellin transformation is not convergent due to bad UV behaviour of gravitation scattering amplitudes.

4d Scattering: Modified Mellin Transformation

Define: $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{-i\epsilon\omega u} a(\epsilon\omega, z, \bar{z}, \sigma).$

where $a(\epsilon\omega, z, \bar{z}, \sigma)$ is the momentum space (creation) annihilation operator of a massless particle with helicity σ when $(\epsilon = -1)$ $\epsilon = 1$. In terms of these fields we can write

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \langle \prod_{i=1}^{n} \phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i, z_i, \bar{z}_i)\rangle.$$

The field $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})$ transforms under BMS transformations as:

Supertranslation: $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z}) \to \phi^{\epsilon}_{h,\bar{h}}(u + f(z,\bar{z}), z, \bar{z})$

Superrotation: $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z}) \to \left(\frac{dw}{dz}\right)^{h} \left(\frac{d\bar{w}}{d\bar{z}}\right)^{\bar{h}} \phi^{\epsilon}_{h,\bar{h}}(u', z', \bar{z}')$

These are exactly the same as the Carrollian CFT primaries that were defined earlier.

This is a central observation of what is to follow.

Proposal: Scattering Amplitude = Carroll CFT Correlator

It is natural to identify **the time-dependent correlation functions of primary fields in a Carrollian CFT** with **the modified Mellin transformation:**

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_i \langle \phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i, z_i, \bar{z}_i)\rangle.$$

The time-dependent correlators of a 3d Carroll CFT compute the 4d scattering amplitudes in the Mellin basis.

What have we learnt so far?

* Carrollian physics emerges in the vanishing speed of light limit of Lorentzian physics.
* Carrollian CFTs are natural holographic duals of flat spacetimes as they inherit the asymptotic symmetries of the bulk theory.
* Over the years, a lot of evidence has been gathered about especially the duality between 3d flatspace and 2d Carroll CFTs.
* In particular, a BMS-Cardy formula in a 2d Carroll CFT reproduces the entropy of the cosmological horizon of Flatspace Cosmologies, providing one of the most important checks of the holographic analysis in flatspace.
* A stumbling block was the formulation of scattering in Carroll CFTs.

What have we learnt so far?

* The S-matrix is the most important observable for Quantum gravity in flatspace.

* Carroll CFT correlation functions have two branches. One of them is time-independent and gives correlations of a 2d CFT. The other one gives spatial delta functions and depends on the null time direction.

* Using modified Mellin transformations, can show this delta-function branch has the correct properties for reproducing scattering amplitudes in the bulk.

* So scattering amplitudes are connected to Carroll CFT correlations in a rather non-trivial and non-obvious way.

Open questions: Flat Holography

* Why is the "electric" leg important for scattering?
* Going beyond 2 and 3 point functions. 4 point? Can we construct an interacting theory and make the connection concrete? Input from gravity?
* Limit from AdS/CFT for flatspace scattering? Does not seem to work at first sight.
* Bootstrap for Carroll CFT for d>2. [Bootstrap for d=2 (AB, Gary, Zodinmawia 2016)]
* Connection to the picture of Laura, Romain, Adrien.
  Celestial Holography as a "restriction" of Carrollian Holography?
* Addressing the question of S=A/4G for d=4.
* Vacuum degeneracy and memory in Carroll CFTs.

Tensionless Strings

Null Strings?! What? Why?

* Massless point particles move on null geodesics. Worldlines are null.

* Null strings: extended analogues of massless point particles.
  Massless point particles => Tensionless strings.

* Tensionless or null strings: studied since **Schild** in 1970's.

* Tension $T = \dfrac{1}{2\pi\alpha'} \to 0$: point particle limit of string theory => Classical gravity.

* Tensionless regime: $T = \dfrac{1}{2\pi\alpha'} \to \infty$: **ultra-high energy, ultra-quantum gravity!**

Null strings are vital for:

A. Strings at **very high temperatures**: Hagedorn Phase.

B. Strings near **spacetime singularities:** Strings near Black holes, near the Big Bang.

C. Connections to **higher spin theory**.

Summary of Results

* 2d Conformal Carrollian (or BMS3) and its supersymmetric cousins arise on the worldsheet of the tensionless string replacing the two copies of the (super) Virasoro algebra.

* Classical tensionless strings: properties can be derived intrinsically or as a limit of usual tensile strings.

* Quantum tensionless strings: many surprising new results.

Classical Tensionless Strings

Isberg, Lindstrom, Sundborg, Theodoridis 1993

AB 2013; AB, Chakrabortty, Parekh 2015.

Going tensionless

Isberg, Lindstrom, Sundborg,Theodoridis 1993

Start with Nambu-Goto action:

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}}. \tag{1}$$

To take the tensionless limit, first switch to Hamiltonian framework.

- Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.
- Constraints: $P^2 + T^2\gamma\gamma^{00} = 0, \; P_m\partial_\sigma X^m = 0$.
- Hamiltonian: $\mathcal{H}_T = \mathcal{H}_\mathcal{C} + \rho^i(\text{constraints})_i = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho\, P_m\partial_\sigma X^m$.

Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi\, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho\dot{X}^m\partial_\sigma X_m + \rho^2\partial_\sigma X^m\partial_\sigma X_m - 4\lambda^2T^2\gamma\gamma^{00}\right] \tag{2}$$

Identifying

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2T^2 \end{pmatrix},$$

action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}g^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n\eta_{mn}. \tag{3}$$

Going Tensionless ...

Isberg, Lindstrom, Sundborg, Theodoridis 1993

- Tensionless limit can now be taken systematically.
- $T \to 0 \Rightarrow$

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 \end{pmatrix}.$$

- Metric is degenerate. $\det g = 0$.
- Replace degenerate metric density $T\sqrt{-g}g^{\alpha\beta}$ by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density

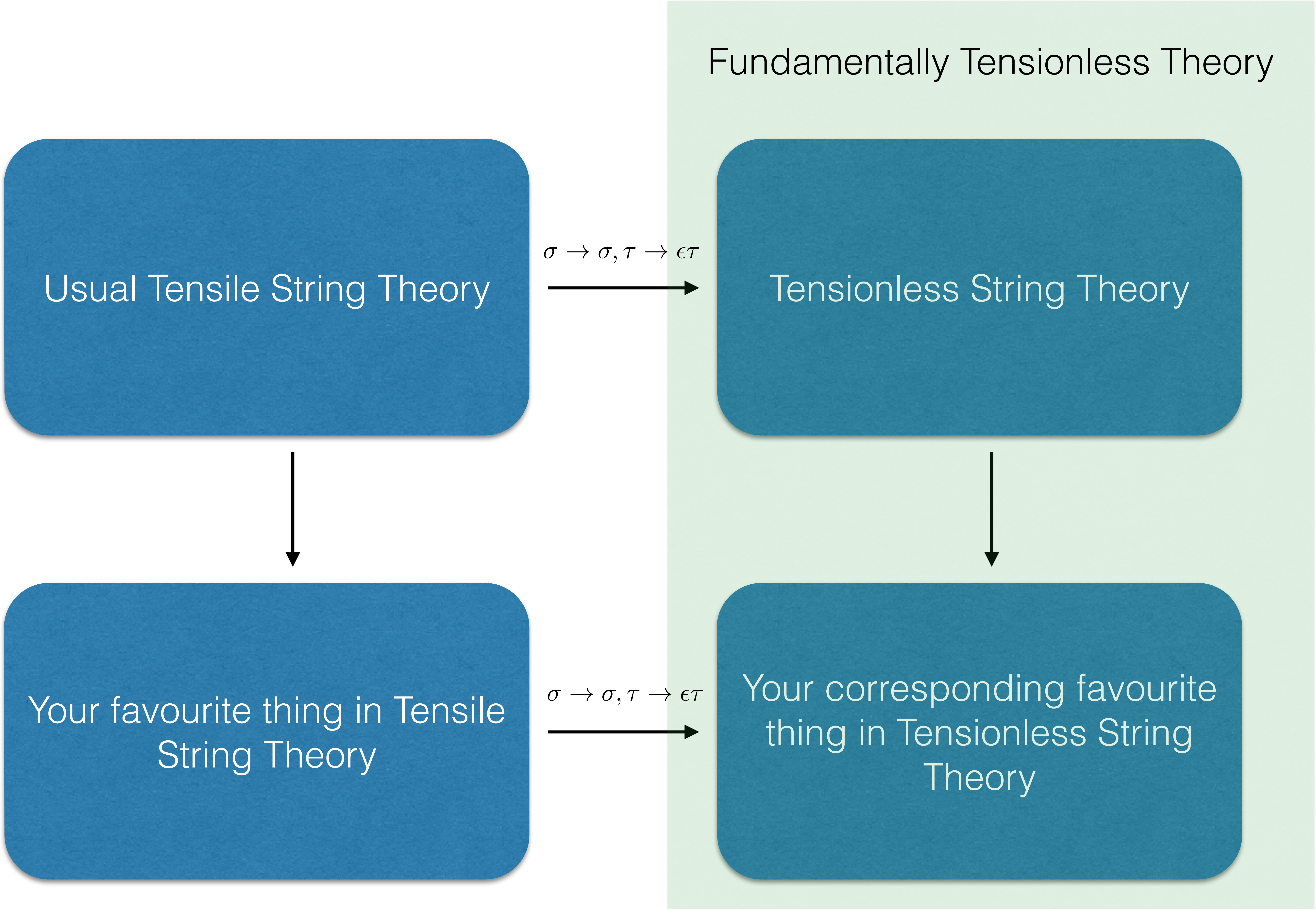
$$V^\alpha \equiv \frac{1}{\sqrt{2\lambda}}(1, \rho) \tag{4}$$

- Action in $T \to 0$ limit

$$\boxed{S = \int d^2\xi \;\; V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}.} \tag{5}$$

- Starting point of tensionless strings.
- Need not refer to any parent theory. Treat this as action of fundamental objects.

Completing the square?



Gauge and Residual Gauge Symmetries

Tensionless action is invariant under world-sheet diffeomorphisms.

Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir. Central to understanding string theory.

Tensionless: Similar residual symmetry left over after gauge fixing.

For world-sheet diffeomorphism: $\xi^\alpha \to \xi^\alpha + \varepsilon^\alpha$, change in vector density: $\delta_\varepsilon V^\alpha = -V \cdot \partial \varepsilon^\alpha + \varepsilon \cdot \partial V^\alpha + \frac{1}{2}(\partial \cdot \varepsilon) V^\alpha$

Tensionless residual symmetries: for $V^\alpha = (v, 0)$, $\varepsilon^\alpha = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$

Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma$, $M(g) = g(\sigma)\partial_\tau$. Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$

$$L(f) = \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = \sum_n a_n L_n, \quad M(g) = \sum_n b_n e^{in\sigma}\partial_\tau = \sum_n b_n M_n.$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12}(m^3 - m)\delta_{m+n,0}, \quad [M_m, M_n] = 0.$$
$$[L_m, M_n] = (m-n)M_{m+n} + \frac{c_M}{12}(m^3 - m)\delta_{m+n,0}.$$

Tensionless Limit from the Worldsheet

A Bagchi 2013

- Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$:

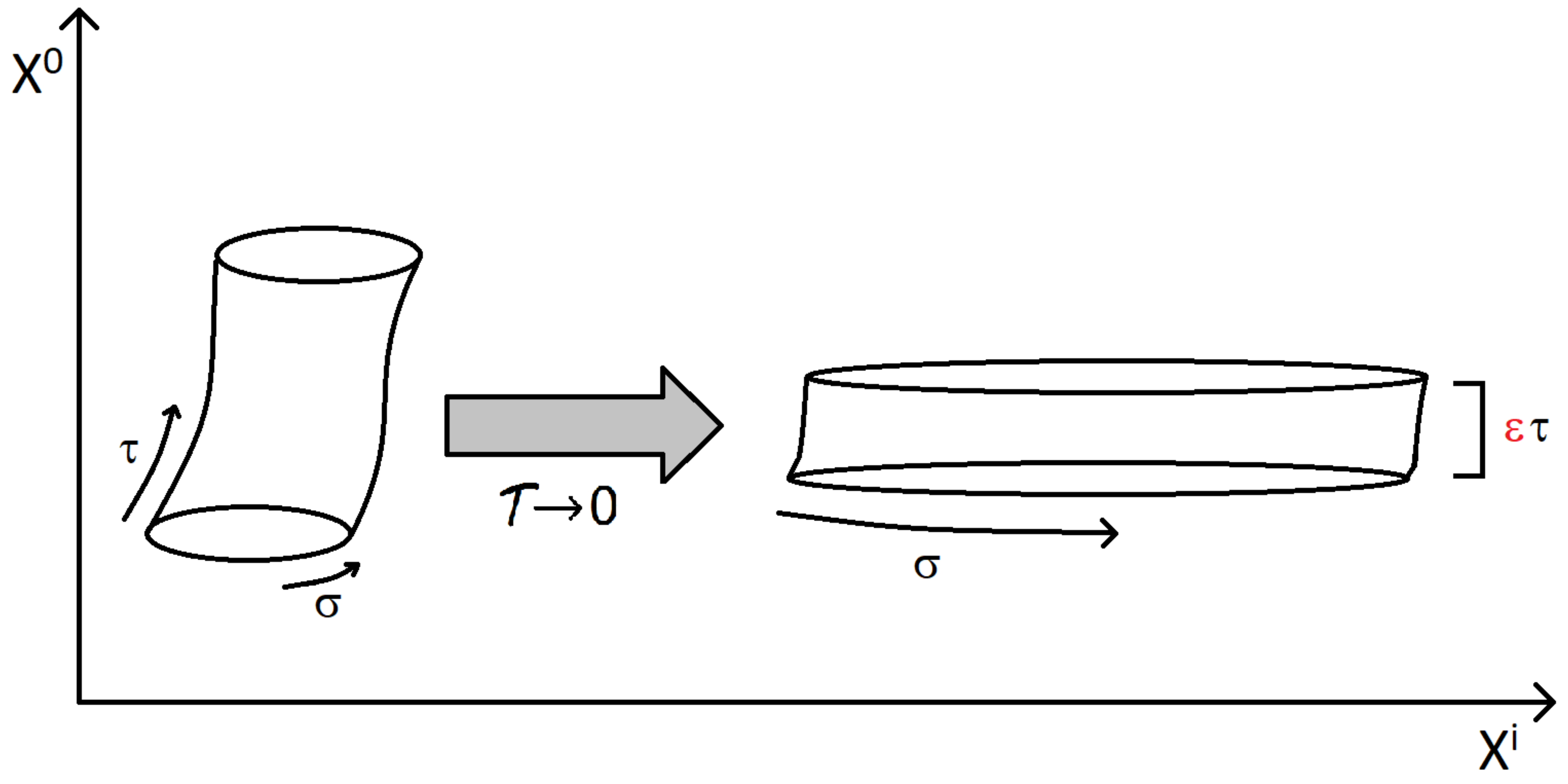
$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0}$$

- World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder.

$$\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}}$$

  where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

- Tensionless limit $\Rightarrow$ length of string becomes infinite ($\sigma \to \infty$).
- Ends of closed string identified $\Rightarrow$ limit best viewed as ($\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$).



Tensionless Limit from the Worldsheet

A Bagchi 2013

- Define

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}).$$

- New vector fields (L_n, M_n) well-defined in limit and given by:

$$L_n = ie^{in\sigma}(\partial_\sigma + in\tau\partial_\tau), \quad M_n = ie^{in\sigma}\partial_\tau.$$

- These are *exactly the generators defined previously* . Close to form BMS_3.

$$[L_m, L_n] = (m-n)L_{m+n} \quad [L_m, M_n] = (m-n)M_{m+n} \quad [M_m, M_n] = 0.$$

- Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$
- Worldsheet velocities $v = \frac{\sigma}{\tau} \to \infty$. Effectively, $\frac{v}{c} \to \infty$
- Hence worldsheet speed of light $\to 0$. Carrollian limit.

- Degenerate worldsheet metric.
- Riemannian tensile worldsheet $\to$ Carrollian tensionless worldsheet.

Tensionless EM Tensor and constraints

A Bagchi 2013

Spectrum of tensile string theory (in conformal gauge in flat space)

- Quantise worldsheet theory as a theory free scalar fields.
- Constraint: vanishing of EOM of metric (which is fixed to be flat).
- Op form: Physical states vanish under action of modes of E-M tensor.

EM tensor for 2d CFT on cylinder: $$T_{cyl} = z^2 T_{plane} - \frac{c}{24} = \sum_n \mathcal{L}_n e^{in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_n \bar{\mathcal{L}}_n e^{in\bar{\omega}} - \frac{\bar{c}}{24}$$

$$T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \bar{T}_{cyl} \right) = \sum_n (L_n - in\tau M_n) e^{in\sigma} - \frac{c_L}{24}$$

Ultra-relativistic EM tensor

$$T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \bar{T}_{cyl} \right) = \sum_n M_n e^{in\sigma} - \frac{c_M}{24}$$

- Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

- Quantum version: physical spectrum of tensionless strings restricted by

$$\langle \text{phys} | T_{(1)} | \text{phys}' \rangle = 0, \quad \langle \text{phys} | T_{(2)} | \text{phys}' \rangle = 0.$$

Intrinsic Analysis: EOM and Mode Expansions

AB, Chakrabortty, Parekh 2015

- Equation of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^\mu = 0$.
- Solution: $X^\mu(\sigma,\tau) = x^\mu + \sqrt{2c'}A_0^\mu \sigma + \sqrt{2c'}B_0^\mu \tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left(A_n^\mu - in\tau B_n^\mu\right) e^{in\sigma}$
- Closed string b.c.: $X^\mu(\sigma,\tau) = X^\mu(\sigma+2\pi,\tau) \Rightarrow A_0^\mu = 0$.
- Constraints: $\dot{X}^2 = 2c'\sum_{m,n} B_{-m}\cdot B_{m+n}\, e^{in\sigma} = 0, \quad \dot{X}\cdot X' = 2c'\sum_{m,n}(A_{-m} - in\tau B_{-m})\cdot B_{m+n}\, e^{in\sigma} = 0$
- Define: $L_n = \sum_m A_{-m}\cdot B_{m+n}, \quad M_n = \sum_m B_{-m}\cdot B_{m+n}$
- Classical constraints in terms of modes: $\sum_n (L_n - in\tau M_n)\; e^{in\sigma} = 0 = T_{(1)}, \quad \sum_n M_n\, e^{in\sigma} = 0 = T_{(2)}.$

  Familiar form obtained earlier from purely algebraic considerations.
- The algebra of the modes $\{A_m^\mu, A_n^\nu\} = 0, \quad \{B_m^\mu, B_n^\nu\} = 0, \quad \{A_m^\mu, B_n^\nu\} = -im\delta_{m+n,0}\,\eta^{\mu\nu}.$
- The worldsheet symmetry algebra of tensionless strings, now constructed from the quadratics of the modes:

$$\{L_m, L_n\} = -i(m-n)L_{m+n}, \; \{L_m, M_n\} = -i(m-n)M_{m+n}, \; \{M_m, M_n\} = 0.$$

Quantization: $\{,\}_{PB} \to -\frac{i}{\hbar}[,]$ leads to the BMS_3 Algebra.

Limiting Analysis: EOM and Mode Expansions

AB, Chakrabortty, Parekh 2015

- Tensile string mode expansion: $X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{2\alpha'}\alpha_0^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)} + \alpha_n^\mu e^{-in(\tau-\sigma)}].$
- The limiting procedure: $\tau \to \epsilon\tau,\ \sigma \to \sigma,\ \alpha' = c'/\epsilon$ with $\epsilon \to 0$

$$X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{\frac{2c'}{\epsilon}}\alpha_0^\mu\epsilon\tau + i\sqrt{\frac{2c'}{\epsilon}}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in\sigma}(1-in\epsilon\tau) + \alpha_n^\mu e^{in\sigma}(1-in\epsilon\tau)],$$

$$= x^\mu + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^\mu\tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left[\frac{\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)\right]e^{in\sigma}.$$

- Thus we get a relation between the tensionless and tensile modes:

$$A_n^\mu = \frac{1}{\sqrt{\epsilon}}(\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu), \quad B_n^\mu = \sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu).$$

- The equivalent of the Virasoro contraints

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left[\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right]$$

Quantum Tensionless Strings

A summary of quantum results

* Novel closed to open string transition as the tension goes to zero.
  [AB, Banerjee, Parekh (PRL) 2019]

* Careful canonical quantisation leads to not one, but <u>three different vacua</u> which give rise to different quantum mechanical theories arising out of the same classical theory.
  [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]

* Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction on the other theory. All acceptable limits of quantum tensile strings.
  [AB, Mandlik, Sharma 2021]

* Interpretation in terms of Rindler physics on the worldsheet.
  [AB, Banerjee, Chakrabortty (PRL) 2021]

* Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- From a single classical theory, several inequivalent quantum theories may emerge. This happens when we consider canonical quantisation of tensionless string theories.

- As we saw earlier Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

  Quantum version: physical spectrum of tensionless strings restricted by $\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0.$

- This amounts to $\langle phys|L_n|phys'\rangle = 0, \quad \langle phys|M_n|phys'\rangle = 0.$

- For each type of oscillator F obeying $\langle phys|F_n|phys'\rangle = 0$, there can be three types of solutions.

$$1. \quad F_n|phys\rangle = 0 \quad (n > 0),$$

$$2. \quad F_n|phys\rangle = 0 \quad (n \neq 0),$$

$$3. \quad F_n|phys\rangle \neq 0, \ but \ \langle phys'|F_n|phys\rangle = 0.$$

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- Here $F_n = (L_n, M_n)$. Hence seemingly nine conditions:

$$L_m|phys\rangle = 0,\ (m>0),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix};\ L_m|phys\rangle = 0,\ (m\neq 0),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix};\ L_m|phys\rangle \neq 0,\ (\forall\ m),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix}$$

- But the underlying BMS algebra also has to be satisfied. It turns out that only three of the nine choices lead to consistent solutions.

- These are three inequivalent vacua, leading to three inequivalent quantum theories.

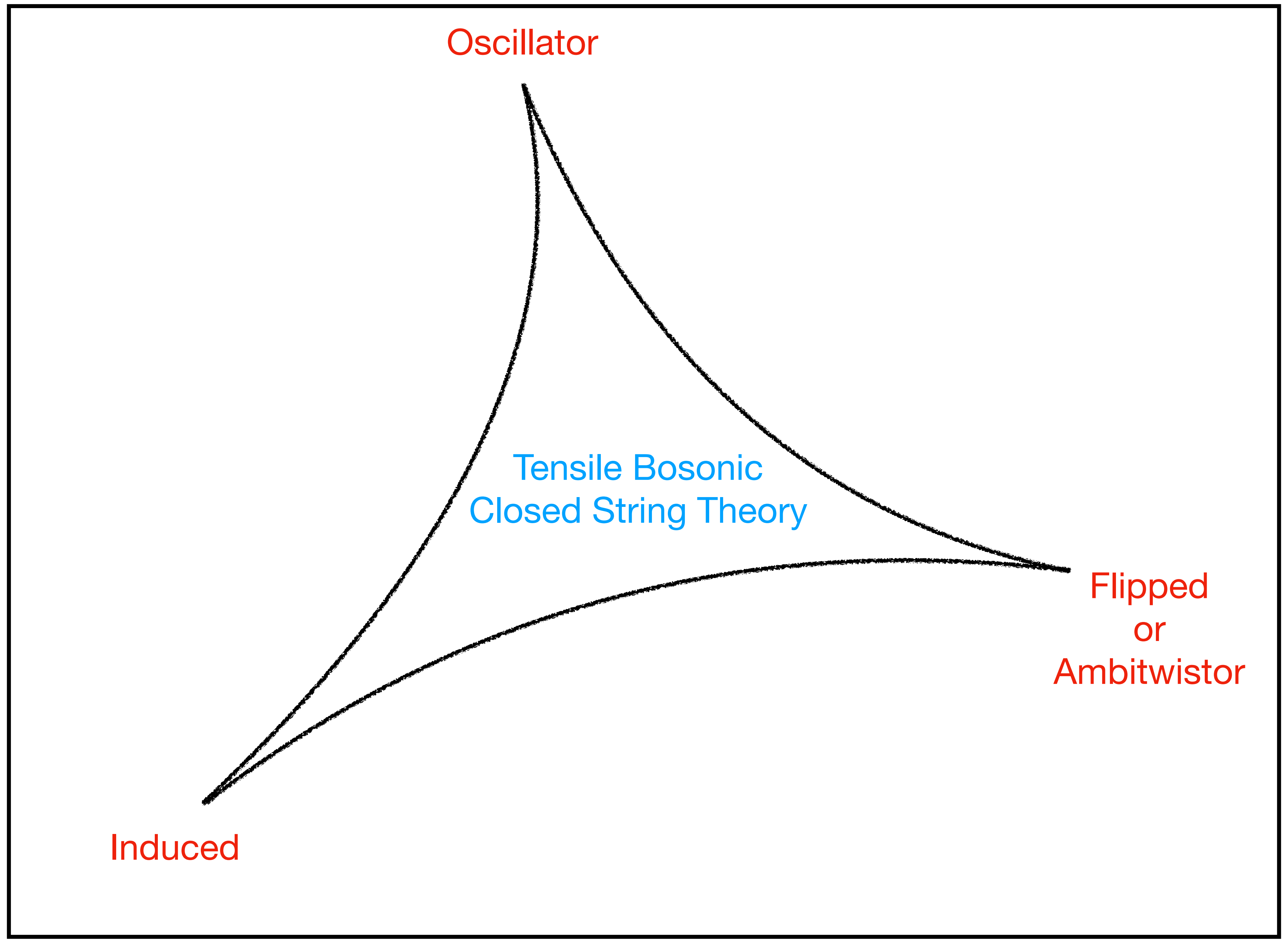
  - **Induced vacuum:** Theory obtained from the limit of usual tensile strings.

  - **Flipped vacuum:** Leads to ambitwistor strings. (See e.g. Casali, Tourkine, (Herfray) 2016-17)

  - **Oscillator vacuum:** Interesting new vacuum. Contains hints of huge underlying gauge symmetry.

Critical Dimensions

AB, Mandlik, Sharma. 2105.09682



Tensionless corners of Quantum Tensile String Theory

Other results

* Tensionless superstrings: Two varieties depending on the underlying Superconformal Carrollian algebra.

* Homogeneous Tensionless Superstrings: Fermions scale in same way. Previous construction: Lindstrom, Sundborg, Theodoridis 1991. Limiting point of view: AB, Chakrabortty, Parekh 2016.

* Inhomogeneous Tensionless Superstrings: Fermions scale differently. New tensionless string! AB, Banerjee, Chakrabortty, Parekh 2017-18.

* Possible counting of BTZ microstates with winding null strings on the horizon. AB, Grumiller, Sheikh-Jabbari (in progress)

Open questions: Tensionless Strings

* Analogous calculation of beta-function=0. Consistent backgrounds?
* Linking up to Gross-Mende high energy string scattering from worldsheet symmetries.
* Attacking the Hagedorn transition from the Carroll perspective. Emergent degrees of freedom?
* Strings near black holes, strings falling into black holes?
* Extend “Tale of Three” to superstrings. Different superstring theories?
* Intricate web of tensionless superstring dualities?

What else is cooking?

Carroll Fermions

AB, Basu, Islam, Mondal (d>2, work in progress)

AB, Banerjee, Dutta, Mondal (d=2, work in progress)

Carroll Clifford algebra

* Two metrics for (flat) Carrollian theories: $\eta_{\mu\nu} \to \tilde{h}_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & I_{d-1} \end{pmatrix}, \; -c^2\eta^{\mu\nu} \to \Theta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0_{d-1} \end{pmatrix}$
* Two different Clifford algebra?

$$\boxed{\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2\tilde{h}_{\mu\nu}, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2\Theta^{\mu\nu}}$$

* Both consistent?
* $\tilde{\Sigma}_{ab} \equiv \frac{1}{4}[\tilde{\gamma}_a, \tilde{\gamma}_b]$: should obey the equivalent of the Lorentz algebra, i.e. the algebra of Carroll boosts and rotations.
* Lower gammas do this.
* For upper gammas: rotation matrices are identically zero. Not a faithful representation of the algebra.

Theory with Lower Gammas

* Can build higher dimensional gammas from lower dimensional ones.
* Adjoint of a Carroll spinor: $\bar{\Psi} = \Psi^\dagger \Lambda, \quad \text{where} \quad \Lambda = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
* Defined Λ such that $\bar{\Psi}\Psi$ is a Carroll scalar.
* An action with lower gammas: $S_{\text{lower}} = \int dt d^3x \big(\bar{\Psi} \tilde{\gamma}_0 \partial_t \Psi - m \bar{\Psi}\Psi \big)$
* Carroll symmetry manifest.
* Massless case: Conformal Carroll. Has infinite dimensional supertranslations as well. (Carroll version of Coriolis algebra)
* Possible applications to magic superconductivity in 2+1 dimensional graphene.

Theory with Upper Gammas

* Also can construct higher dimensional gammas from lower dimensional ones.
* One can use Upper Gammas to construct theories for d=2, since we don't have rotations here.
* Action: $S^{2d}_{\text{upper}} = \int dtdx \, \left(\bar{\Psi}\tilde{\gamma}^0\partial_t\Psi + \bar{\Psi}\tilde{\gamma}^1\partial_x\Psi\right)$
* Relations to earlier actions for the tensionless superstrings.
* BMS3 arises as symmetries.

Confusions with Fermions

* Why do the upper gammas not make sense for d>2?

* Carroll Clifford algebras look like "electric" and "magnetic" ones arising from the original Clifford algebra.

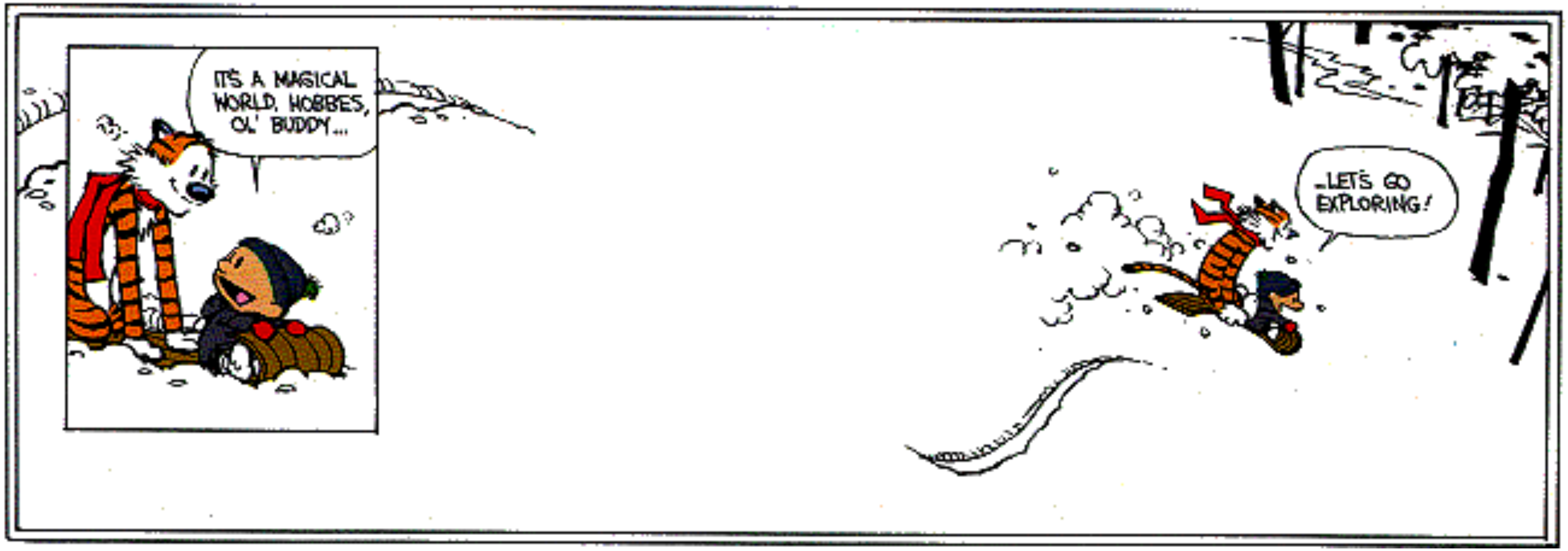
$$\Gamma = \gamma_{lower} + c\gamma_{upper} + \dots$$

* The naive limit c-> 0 does not seem to make sense.

$$\{\Gamma, \Gamma\} = 2g = 2(g^0 + c^2 g^1 + \dots) \Rightarrow \{\gamma_{lower}, \gamma_{upper}\} = 0$$

But this does not work in the explicit representations we have.

* Clearly a lot to understand about Carroll fermions.



Thank you!