PERIMETER INSTITUTE

Symmetries at Null Boundaries

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Motivations

Symmetries at null boundaries with leakyness

- What is the maximal number of symmetries that can be switched on null surfaces ?
- Which coordinates system realize them?
- What is the phase space and how is it organized ?
- How to deal with flux or/and non integrable charges?

- Finite surfaces but, up to renormalizable technics, can be applied to asymptotic boundaries

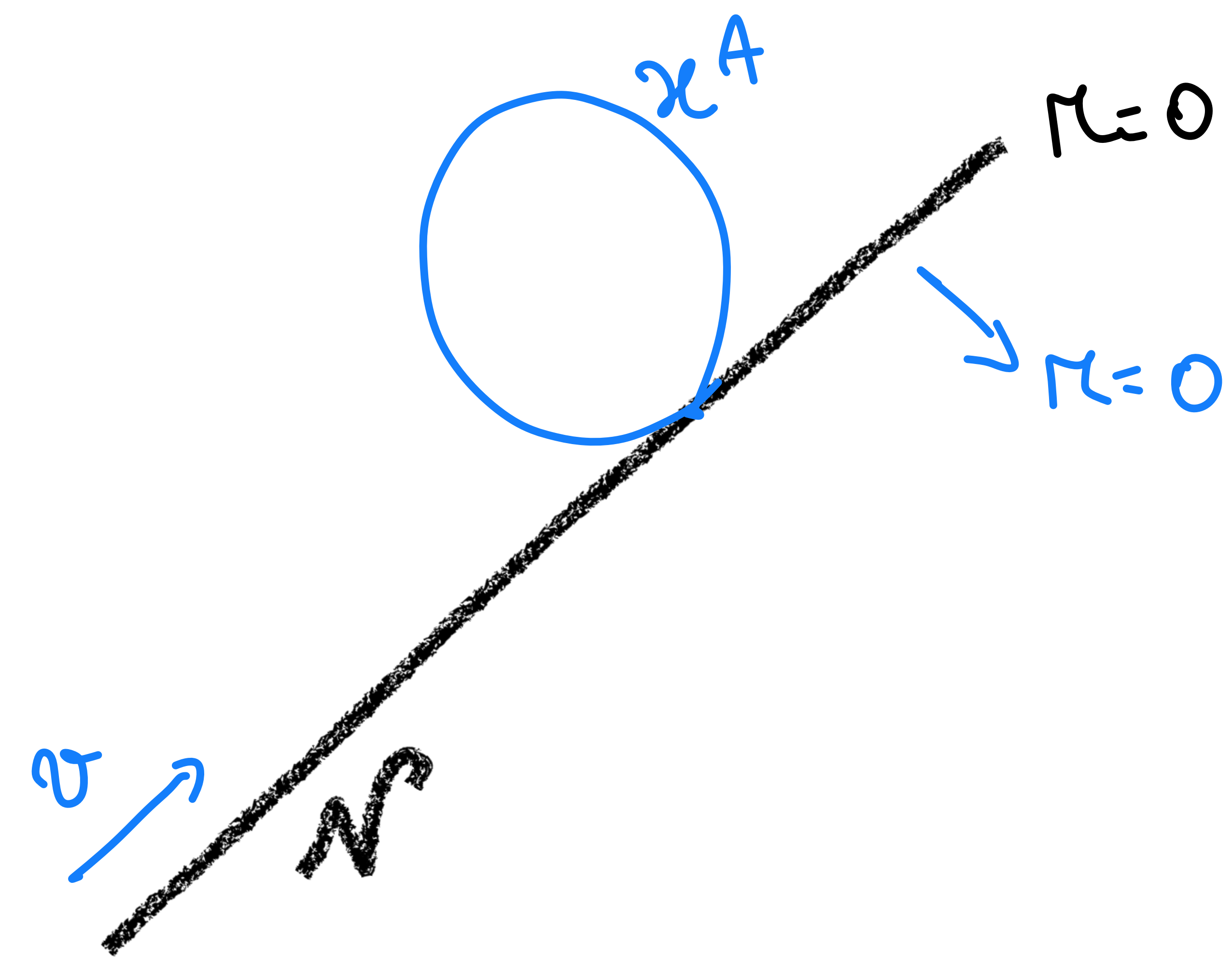
Set-up: Einstein theory & GNC

advanced time

Null surface (v, r, x^A) ; $A = 1 \dots D-2$

transverse

radial



- $g_{rr} = 0$; $g_{rA} = 0$
- $g_{vr} = \eta(v, x^A)$
- Taylor expansion for $g_{AB} = \Omega_{AB}(v, x^A) + O(r)$

$$ds^2 = -V\, dv^2 + 2\eta\, dv\, dr + g_{AB}(dx^A + U^A dv)(dx^B + U^B dv)$$

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$$g_{AB}(v,r,x^A) = \Omega_{AB}(v,x^A) + \mathcal{O}(r)$$

EOM

$$V(v,r,x^A) = V_0 + 2(\eta\kappa - D_v\eta)\,r + \mathcal{O}(r^2)$$

$$U^A(v,r,x^A) = \mathcal{U}^A - \frac{\eta}{\Omega}\Upsilon^A r + \mathcal{O}(r^2)$$

$+ \; V_0 = 0$ to have a null surface

$D_v = \partial_v - \mathcal{L}_{\mathcal{U}}$

$\Omega = \sqrt{\det \Omega_{AB}}$

$\Theta =$ expansion $= \frac{D_v \Omega}{\Omega}$

$N_{AB} =$ shear

$= \frac{1}{2} D_v \Omega_{AB} - \frac{\Theta}{D-2}\Omega_{AB}$

encodes radiation

Constraints (at leading order)

$$D_v\Theta - \kappa\Theta + \frac{1}{D-2}\Theta^2 + N_{AB}N^{AB} = 0$$

$$D_v\left(\Upsilon_A + \Omega\frac{\partial_A \eta}{\eta}\right) - 2\Omega\,\partial_A\left(\kappa + \frac{D-3}{D-2}\Theta\right) + 2\Omega\,\nabla^B N_{AB} = 0$$

Symmetries

$$\xi = T(v, x^A)\,\partial_v + \big(r\,(\mathcal{D}_v T - W(v, x^A)) + O(r^2) \big)\partial_r + \big(Y^A(v, x^A) + O(r) \big)\partial_A$$

$\hookrightarrow$ supertranslation

$\hookrightarrow$ Weyl scaling

$\hookrightarrow$ superrotations

Modified Lie bracket $\quad [\xi_1, \xi_2]_* = [\xi_1, \xi_2] - \delta_{\xi_1}\xi_2 + \delta_{\xi_2}\xi_1$

Assuming $\quad \delta_\xi T = \delta_\xi W = \delta_\xi Y = 0$

Symmetry algebra $\quad \mathrm{Diff}(\mathcal{N}) \oplus \mathrm{Weyl}(\mathcal{N})$

Relations to the literature

- Boundary conditions restrictions $\eta = 1$ [Donnay, Giribet, González, Pino]

$$\delta_\xi \eta = 2\eta D_v T + T \partial_v \eta - W\eta + Y^A \partial_A \eta \longrightarrow \delta_\xi \eta \big|_{\eta=1} = 0 \iff W = \frac{D_v T}{2}$$

- Subalgebra = BMSW [Freidel, Oliveri, Pranzetti, Speziale]

$$T = T_0(x^A) + v\,\frac{W_0(x^A)}{2}$$
$$W = W_0(x^A)$$
$$Y = Y(x^A)$$

$$(\mathrm{Diff}(S) \ltimes \mathbb{R}) \ltimes \mathbb{R}$$

- Extension to radial translation [Ciambelli, Leigh]

$$(\mathrm{Diff}(N) \ltimes \mathbb{R}) \ltimes \mathbb{R}$$

Charges

- Open system (leakyness) -> non-integrable charge and flux balance laws

$$\dot{Q}^I = -F$$

- Conjecture: Charges can be made integrable by a change of slicing whenever there is no propagating degrees of freedom passing through the surface

$$\xi \text{ s.t. } \delta\xi = 0 \longrightarrow \tilde{\xi} = f(\xi) \text{ s.t. } \delta\tilde{\xi} = 0$$

FLUX = FAKE + GENUINE → dynamical dof (eg graviton)

↳ can be reabsorbed by a change of slicing

Charges $\left(U^A = 0 , \; \Upsilon_A = 0 \right)$

$$16\pi G \, \delta\!\!\!/ Q_\xi = \int d^{D-2}x \left(W \, \delta\Omega + T \left(-2\Omega \, \delta\Theta + \Omega \Theta \frac{\delta\eta}{\eta} \right) - \Gamma(K) \, \delta\Omega - \Omega N^{AB} \delta\Omega_{AB} \right)$$

Case N_AB=0 ; for example 3D

$$\tilde{W} = W - \Gamma(K) T$$

$$\tilde{T} = \Omega \Theta T$$

$$16\pi G \, \delta\!\!\!/ Q = \int \tilde{W} \, \delta\Omega + \tilde{T} \, \delta P \qquad P = \ln \eta/\Theta^2$$

$\rightarrow$ Integrable for $\delta\tilde{W} = \delta\tilde{T} = 0$

Algebra: $\delta_\xi \Omega = \tilde{T}$; $\delta_\xi P = -\tilde{W}$ $\longrightarrow$

HEISENBERG ALGEBRA

$$\{ \Omega(v,x) , P(v',x') \} = 16\pi G \, \delta^{(2)}(x - x')$$

Generic case

$$16\pi G \, \cancel{\delta} Q_\xi = \int d^{D-2}x \left(W \delta\sqrt{\Omega} + T\left(-2\sqrt{\Omega}\,\delta\Theta + \sqrt{\Omega}\,\Theta\,\frac{\delta\eta}{\eta} - \Gamma(K)\,\delta\sqrt{\Omega} - \sqrt{\Omega}\,N^{AB}\delta\Omega_{AB}\right)\right)$$

$$= \int d^{D-2}x \left(\tilde{W}\,\delta\sqrt{\Omega} + \tilde{T}\,\delta P - T\sqrt{\Omega}\,\underbrace{N^{AB}\,\delta\Omega_{AB}} \right)$$

N_{AB} is traceless

→ no way to absorb that term

N_{AB} encodes the radiation (gravitons passing through $\mathcal{N}$)

Using the modified bracket [Barnich/Troessaert] $\delta_{\xi_2} Q^I_{\xi_1} = Q^I_{[\xi_1,\xi_2]} + K_{\xi_1,\xi_2} - F_{\xi_2}(\delta_{\xi_1} g)$

- Thermodynamical slicing $\delta T = \delta W = \delta Y = 0$ [Adami, Sheikh-Jabbari, Taghiloo, Yavartanoo]

$$Q^I = \int W \, \Omega + Y^A \Upsilon_A + T(-\Gamma\Omega + U^A \Upsilon_A)$$

$$F = \int T \Big(\underbrace{-2\Omega\,\delta\Theta + \Omega\,\Theta\,\tfrac{\delta\eta}{\eta} + \Omega\,\delta\Gamma - \Upsilon_A\,\delta U^A}_{\text{FAKE}} \underbrace{- \Omega N^{AB}\,\delta\Omega_{AB}}_{\text{GENUINE}} \Big)$$

$$\frac{dQ^I}{dt} = -\left(F_{FAKE} + F_{GEN}\right)$$

Diff(N) ⊕ Weyl w/ $K_{\xi} = 0$

- Genuine slicing $\delta \tilde{T} = \delta \tilde{W} = 0$

$$Q^I = \int \tilde{W} \, \Omega + \tilde{T}\, P + Y^A \Upsilon_A$$

$$F = \int \underbrace{T \left(-\Omega N^{AB}\,\delta\Omega_{AB}\right)}_{\text{FAKE}}$$

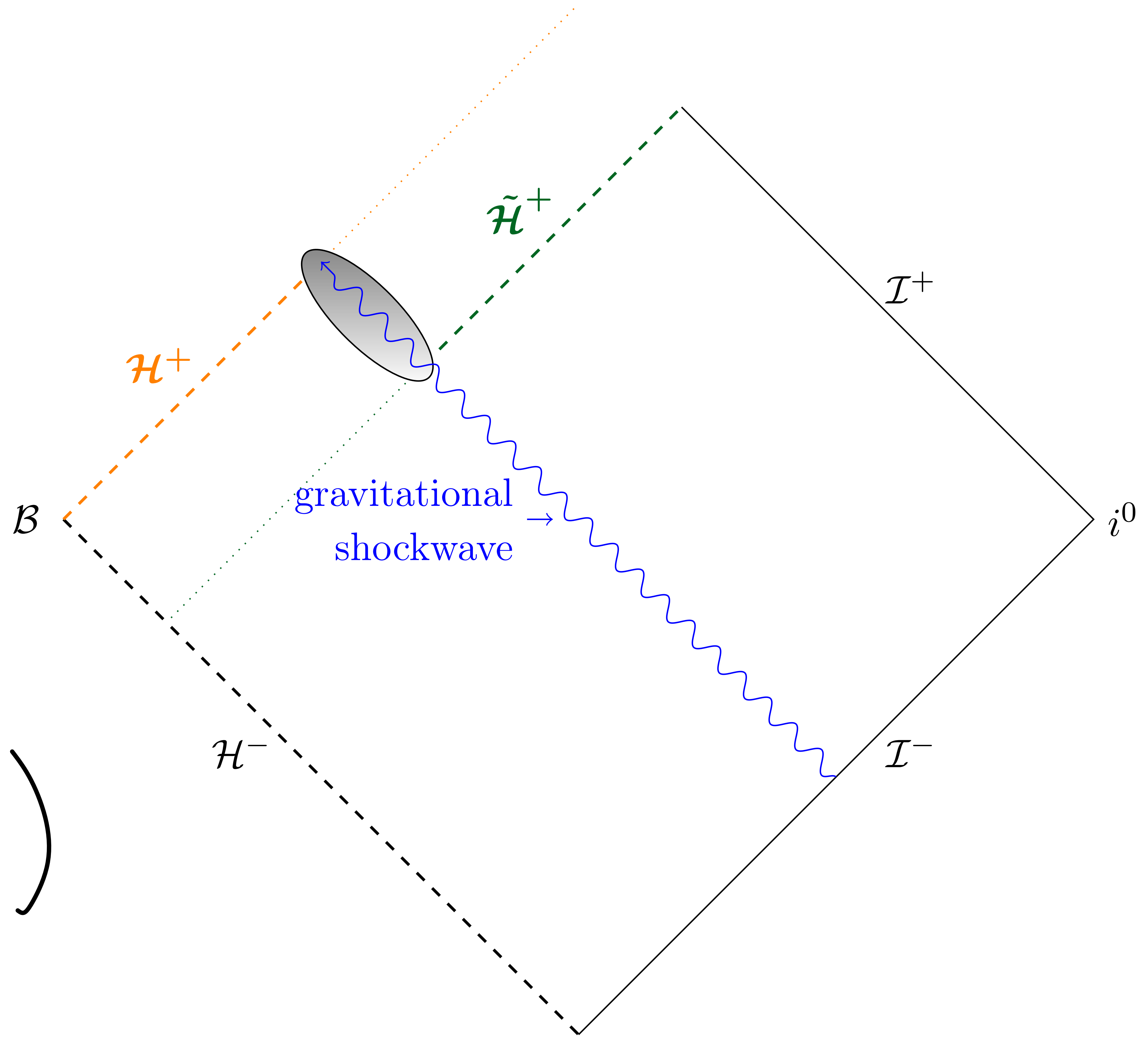
$$\frac{dQ^I}{dt} = -\left(F_{GEN}\right)$$

Heis. ⊕ Diff(S) w/ $K_{\xi} = 0$

Null boundary memory effects

$$\Delta \Omega = \int_{-\infty}^{\infty} dv \, \frac{\Omega}{\kappa} N_{AB} N^{AB}$$

$$\Delta J_A = \int_{-\infty}^{\infty} dv \, 2\Omega \left(\bar{\nabla}_A (\Theta^{-1} N^2) - \nabla^B N_{AB} \right)$$



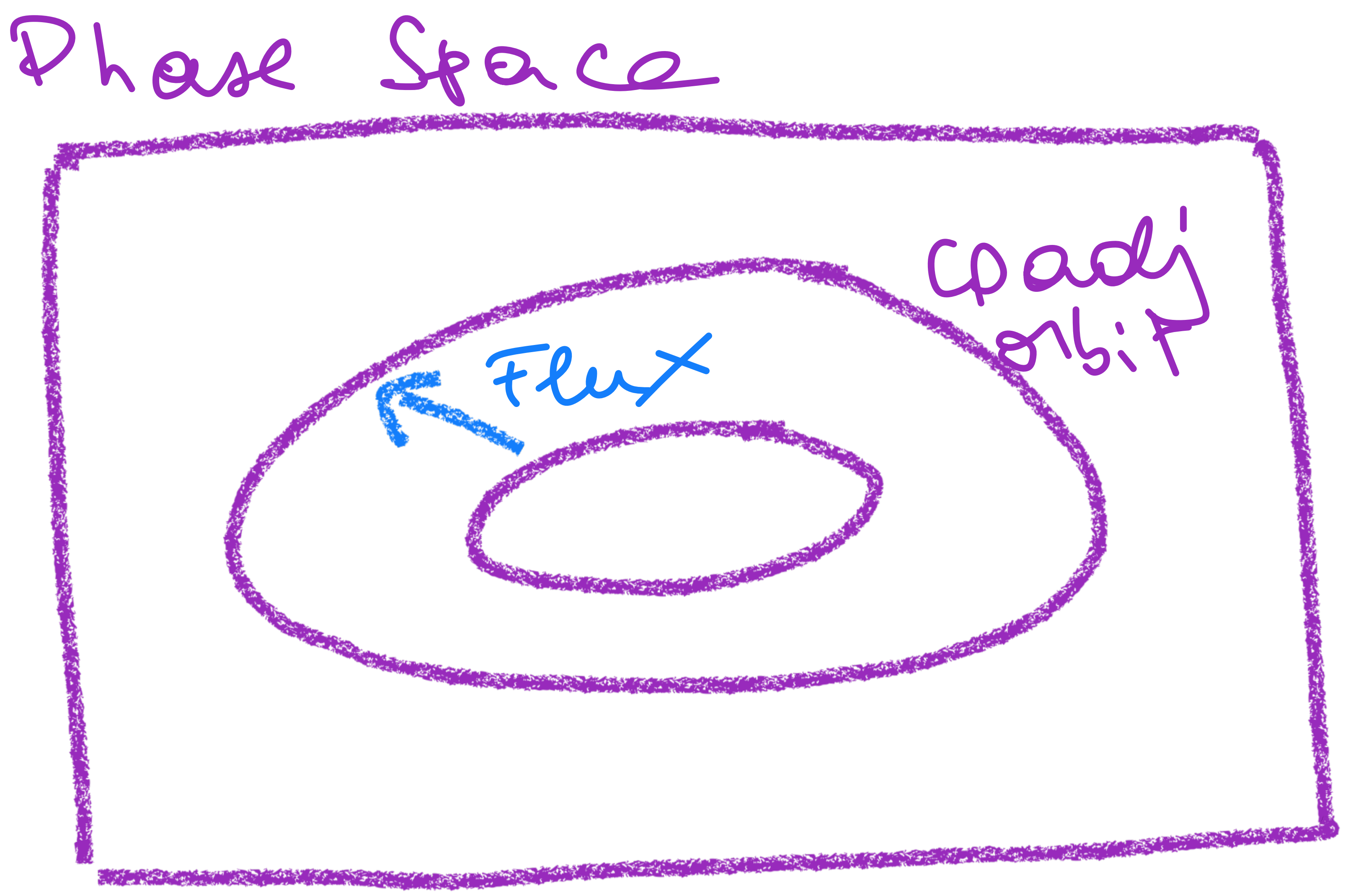
Summary

- What is the maximal number of symmetries that can be switched on null surfaces ?
  - We obtained D codimension 1 functions (unspecified time dependence)

$\mathrm{Diff}(N) \oplus \mathrm{Weyl}$ or $\mathrm{Heis} \oplus \mathrm{Diff}(S)$

- Which coordinates system realize them?
  - Minimize the conditions on the coordinates to get more symmetries

- What is the phase space and how it is organized ?
    - Coadjoint orbits and flux
- How to deal with flux, or apparent non integrable charges?

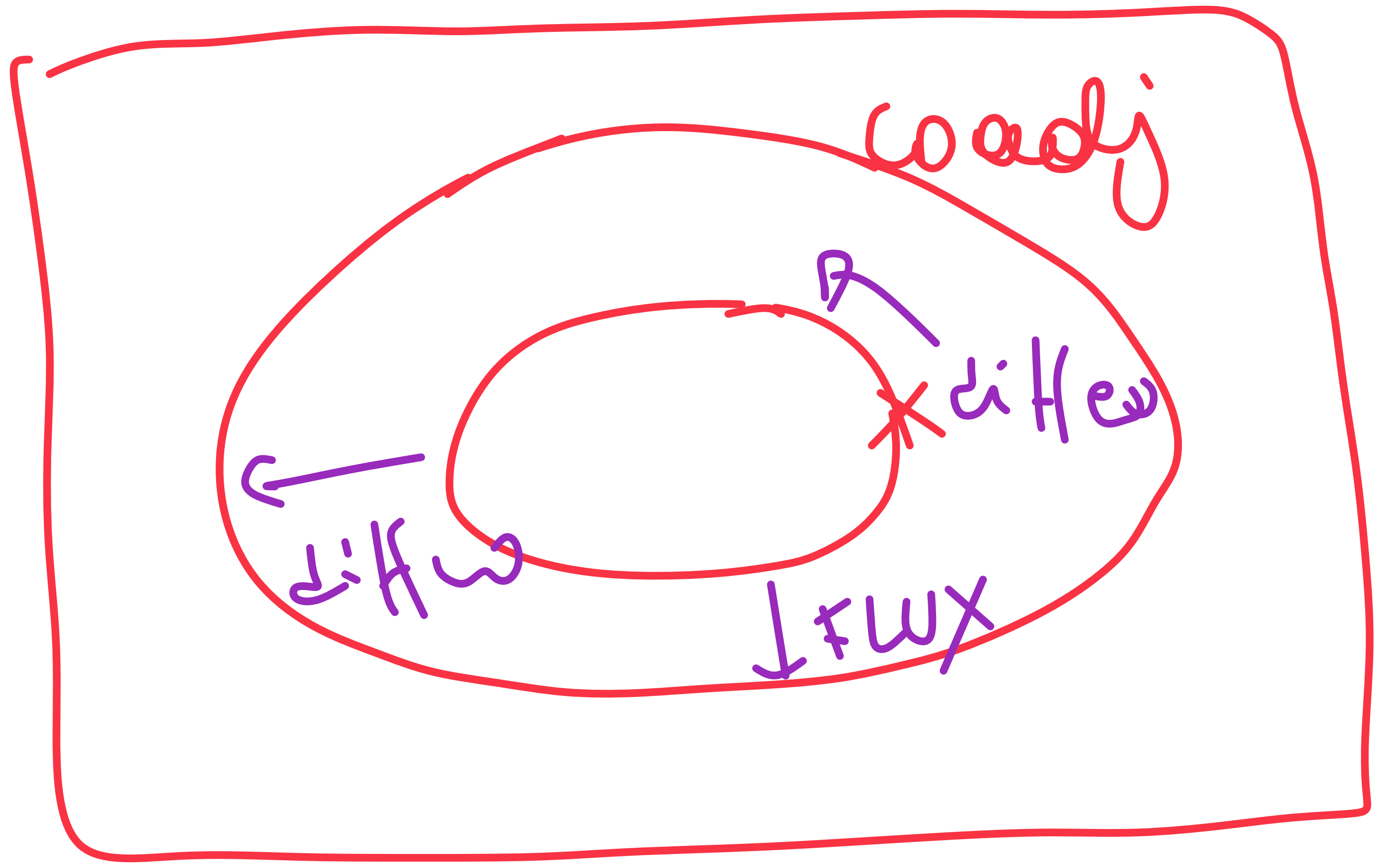


Flux = FAKE + GENUINE

FAKE: eg: switching boundary structure

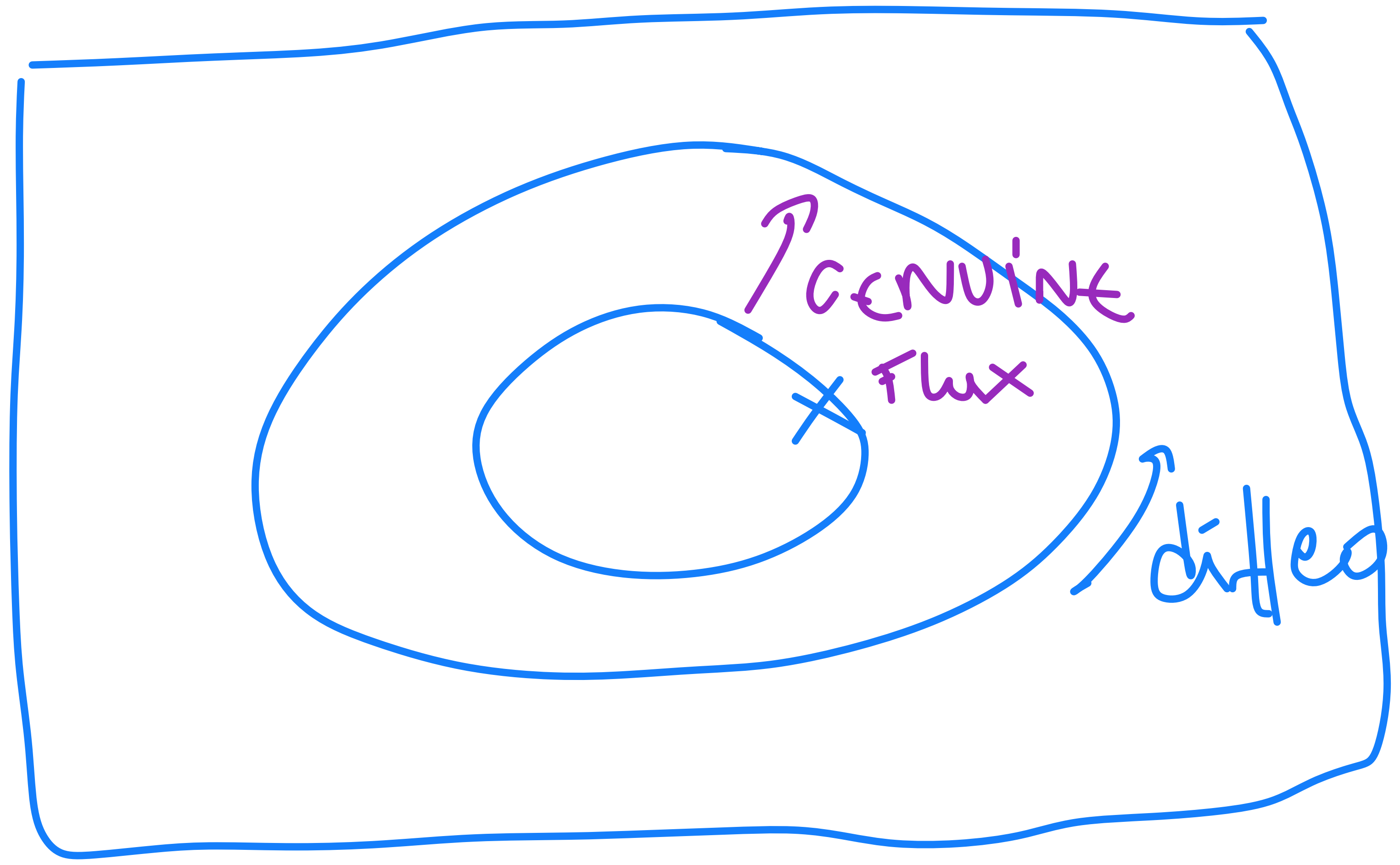
GENUINE: propagating dof.

Thermodynamical slicing



False & genuine

Genuine slicing



Genuine

Future directions

- Still room for bigger symm. , eg switching radial translation
- Allowing time like regim to describe matter falling in the black hole
- Relation to the corner point of view
- Coadjoint orbits
- Heisenberg algebra seems to always appear, how much this group is fundamental?
- …

Danke