Carrollian Fluids and Chthonian versus Celestial Holography

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Highlights

Why fluids?

Solution space of asymptotically locally AdS spacetimes in incomplete Newman–Unti gauge $\rightarrow$ boundary relativistic fluids

Why Carrollian physics?

Asymptotically flat spacetimes $\rightarrow$ Carrollian boundary geometry

What is Carrollian hydrodynamics?

Set of equations obtained

- either from relativistic fluid dynamics at zero light velocity
- or demanding Carrollian diffeomorphism invariance

What is the role of Carrollian fluids in the solution space of Ricci-flat spacetimes?

They carry part of the infinite deep information – unless a "self-duality" condition is imposed

What are the hints about flat holography?

If it exists it should be *Chthonian* rather than *Celestial*

Starring

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Einstein spacetimes with $\Lambda < 0$ in n dimensions

The Newman–Unti gauge for G_{AB} $(r, t, x^i, i = 1, \dots, n-2)$

- Gauge conditions: $G_{rr} = 0$, $G_{rt} = -1$, $G_{ri} = 0$

$$ds^2 = \frac{V}{r}dt^2 - 2dtdr + G_{ij}\left(dx^i - U^i dt\right)\left(dx^j - U^j dt\right)$$

  V, G_{ij}, U^i functions of *all* coordinates

- Residual diffeomorphisms: $\omega(t, \mathbf{x})$, $f(t, \mathbf{x})$, $Y^i(t, \mathbf{x})$
- ASG $\leftrightarrow$ fall-offs/boundary conditions [Compère, Fiorucci, Ruzziconi '19]

Incomplete Newman–Unti gauge fixing [Ciambelli, Marteau, Petropoulos, Ruzziconi '20 – not Céline Zwikel' talk]

- Gauge conditions: $G_{rr} = 0$, $G_{rt} = -1$, $G_{ri} \neq 0$
- Residual diffeomorphisms: $\omega(t, \mathbf{x})$, $f(t, \mathbf{x})$, $Y^i(t, \mathbf{x})$ plus $Z^i(t, \mathbf{x})$, $S_{[ij]}(t, \mathbf{x}) \rightarrow$ extra local $SO(n-2, 1)$

Einstein spacetimes reconstructed

Solution space with incomplete Newman–Unti gauge and mild boundary conditions

- $\frac{n(n-1)+2}{2}$ Einstein's equations $\rightarrow n^2 - 3$ functions of $(t, \mathbf{x})$ $\rightarrow$ boundary data $[\mu, \nu = 0, 1, \ldots, n-2]$
  - $g_{\mu\nu}$ symmetric $\leftarrow \frac{n(n-1)}{2}$
    boundary metric
  - $T_{\mu\nu}$ symmetric and traceless $\leftarrow \frac{n(n-1)}{2} - 1$
    conformal boundary energy–momentum tensor
  - $u^\mu \leftarrow n - 2$
    boundary normalized vector field
- remaining $n-1$ Einstein's equations $\boxed{\nabla_\mu T^{\mu\nu} = 0}$ $\rightarrow$ map to a Weyl-covariant relativistic fluid with velocity u^μ – *linear* trigger for fluid/gravity holographic correspondence

[Bhattacharyya, Hubeny, Minwalla, Rangamani '07; Haack, Yarom '08; etc.]

Relativistic hydrodynamics [Eckart '40; Landau and Lifshitz '60; Israël '87]

- Ignoring matter current and chemical potential
- On arbitrary (non-flat) geometry $g_{\mu\nu}$ of dim $d+1$

$\nabla_\mu T^{\mu\nu} = 0$ plus Gibbs–Duhem & equation of state (conformal)

- $\|\mathrm{u}\|^2 = -k^2 \quad h^{\mu\nu} = g^{\mu\nu} + \frac{u^\mu u^\nu}{k^2}$

$$T^{\mu\nu} = \varepsilon \frac{u^\mu u^\nu}{k^2} + p h^{\mu\nu} + \tau^{\mu\nu} + \frac{u^\mu q^\mu}{k^2} + \frac{u^\nu q^\mu}{k^2}$$

- energy density $\varepsilon = \frac{1}{k^2} T_{\mu\nu} u^\mu u^\nu$ thermodynamic pressure p
- heat current and viscous stress tensor q^μ, $\tau^{\mu\nu}$ – transverse
- fluid velocity u^μ – arbitrary [Eckart '40; Landau and Lifshitz '60]

In $n = 4$ dimensions $\Lambda = -3k^2$

General solution: $6 + 2 + 5$ arbitrary boundary data

- $\mathrm{d}s^2 = -k^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i \mathrm{d}x^j$
- $\mathbf{u} = u_\mu \mathrm{d}x^\mu \to \left\{\sigma^{\mu\nu}, \omega^{\mu\nu}, \mathbf{A} = \frac{1}{k^2}\left(\mathbf{a} - \frac{\Theta}{2}\mathbf{u}\right), \mathscr{D}_\mu\right\}$
- $T_{\mu\nu} \to \{\varepsilon = 2p, q^\mu, \tau^{\mu\nu}\}$ with $\tau^\mu{}_\mu = 0$ & $\nabla_\mu T^{\mu\nu} = 0$
- Cotton $C_{\mu\nu} \to \{c, c^\mu, c^{\mu\nu}\}$ with $c^\mu{}_\mu = 0$ & $\nabla_\mu C^{\mu\nu} = 0$

$$\begin{aligned}\mathrm{d}s^2_{\text{Einstein}} &= 2\frac{\mathbf{u}}{k^2}(\mathrm{d}r + r\mathbf{A}) + r^2\mathrm{d}s^2 - 2\frac{r}{k^2}\sigma_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{\mathrm{S}}{k^4}\\ &+\frac{8\pi G}{k^4 r}\left[\varepsilon \mathbf{u}^2 + \frac{4\mathbf{u}}{3}\left(\mathbf{q} - \frac{1}{8\pi G} * \mathbf{c}\right)\right.\\ &\left.+\frac{2k^2}{3}\left(\boldsymbol{\tau} + \frac{1}{8\pi G k^2} * \boldsymbol{c}\right)\right] + \mathrm{O}\left(1/r^2\right)\end{aligned}$$

$$S_{\mu\nu} = 2u_{(\mu}\mathscr{D}_\lambda\left(\sigma_{\nu)}{}^\lambda + \omega_{\nu)}{}^\lambda\right) - \frac{\mathscr{R}}{2}u_\mu u_\nu + 2\omega_{(\mu}{}^\lambda \sigma_{\nu)\lambda} + (\sigma^2 + k^4\gamma^2)\,h_{\mu\nu}$$

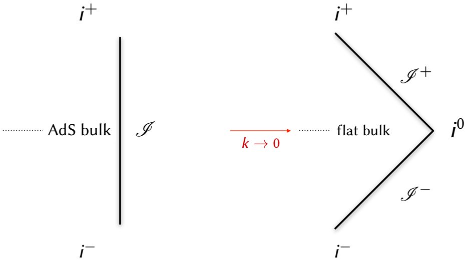
Comments

- The boundary fluid is abstract – no constitutive relations & derivative expansions
- Infinite-dim bulk ASG $\equiv$ boundary-fluid invariance – extra local $SO(n-2,1)$ $\equiv$ hydrodynamic-frame invariance
- Bulk Newman–Unti gauge $\equiv$ boundary fluid with locked velocity – charges $\to$ handle on breaking of $SO(n-2,1)$ [for $n=3$ cf. Campoleoni et al. '19 and talk by Luca Ciambelli]
- A *remarkable* "self-duality" condition $\to$ resummation – u^μ shearless, $q_\mu = \frac{1}{8\pi G} * c_\mu$, $\tau_{\mu\nu} = -\frac{1}{8\pi G k^2} * c_{\mu\nu}$ ($\to$ bulk Weyl)
  $\mathrm{d}s^2_{\text{res. Einstein}} = 2\frac{\mathbf{u}}{k^2}(\mathrm{d}r + r\mathbf{A}) + r^2\mathrm{d}s^2 + \frac{\mathsf{S}}{k^4} + \frac{\mathbf{u}^2}{k^4\rho^2}(8\pi G\varepsilon r + c\gamma)$
  $\rho^2 = r^2 + \frac{1}{2k^4}\omega_{\alpha\beta}\omega^{\alpha\beta} = r^2 + \gamma^2$

A new asymptotic structure

From AdS$_n$ to flat$_n$ asymptotics

$\Lambda = -\frac{(n-1)(n-2)}{2}k^2 \to 0$



Carrollian boundary geometry in $n-1$ dimensions

$k \equiv$ boundary velocity of light $\leftrightarrow$ the boundary $\mathscr{I}$ is *null*

$\to$ Carrollian fluids as boundary data for Ricci-flat bulks

Ideas on Carrollian fluids [ours – complementary views by the Dutch school]

What we *cannot* do: microscopics at $k \to 0$

- no motion is allowed
- no kinetic theory or Boltzmann equation can be constructed
- no thermodynamics

What we can do: take limits and use symmetries

1. Carrollian from relativistic hydrodynamics at $k \to 0$
   - choose a convenient coordinate system
   - assume a behavior for ε, p, q^μ, $\tau^{\mu\nu}$
   - study the limit of $\nabla_\mu T^{\mu\nu} = 0$
2. Momenta & conservation from Carrollian diffeomorphisms
   - effective action invariant under Carrollian diffeomorphisms
   - momenta as variations and conservation equations

A good gauge in $d+1$ dimensions

Riemannian fibration $\mathscr{M}$ *à la* Papapetrou–Randers

$$\mathrm{d}s^2 = -k^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

[cf. discussion in Stefan Vandoren' and Niels Obers' talks about choosing "good" coordinates]

- Carrollian diffeos $t' = t'(t, \mathbf{x})$, $\mathbf{x}' = \mathbf{x}'(\mathbf{x})$ (here $x^0 = kt$)

  reduction of $V^\mu \leftarrow J^\mu_\nu(t,\mathbf{x}) = \frac{\partial x^{\mu\prime}}{\partial x^\nu} = \begin{pmatrix} J(t,\mathbf{x}) & kj_j(t,\mathbf{x}) \\ 0 & J^i_j(\mathbf{x}) \end{pmatrix}$

  - $\frac{1}{\Omega}V_0$ is a Carrollian scalar
  - V^i is a Carrollian vector

- $\mathscr{M} \underset{k\to 0}{\longrightarrow} \mathbb{R}\times\mathscr{S}$ Carrollian geometry with metric

  $\mathrm{d}\ell^2 = a_{ij}\mathrm{d}x^i\mathrm{d}x^j$ & Ehresmann connection $\boldsymbol{e} = \Omega\mathrm{d}t - b_i\mathrm{d}x^i$

  (dual of the kernel of the degenerate metric $\propto \partial_t$)

The Carrollian limit of fluids

Kinematics

$$\mathrm{u} = \gamma \left(-\partial_t + v^i \partial_i\right) \quad \left(u^\mu u_\mu = -k^2\right)$$

with $v^i = \frac{k^2 \Omega \beta^i}{1+k^2 \beta^j b_j}$

- $v^i \underset{k\to 0}{\longrightarrow} 0$ non-trivially
- $\beta^{i\prime} = J^i_j \beta^j$ the *inverse* velocity of Carrollian fluids

Transport

- $\varepsilon \to \varepsilon_{(-1)} k^2 + \varepsilon + \frac{\varepsilon_{(1)}}{k^2} + \frac{\varepsilon_{(2)}}{k^4}$
- similarly for p
- $\frac{q^i}{k^2} \to \pi^i + \frac{1}{k^2} Q^i + \frac{\zeta^i}{k^4}$
- $\tau^{ij} \to -k^2 E^{ij} - \Xi^{ij} - \frac{1}{k^2} \Sigma^{ij} - \frac{\zeta^{ij}}{k^4}$

Three remarks

1. scalings suggested by the AdS boundary relativistic fluids
2. more $\mathrm{O}\left({}^{1}/{k^{2m}}\right)$ terms $\to$ more degrees of freedom
3. more $\mathrm{O}\left({}^{1}/{k^{2m}}\right)$ terms $\to$ more equations

Comparison with the Galilean limit $k \to \infty$

- $\varepsilon \to e\varrho + k^2\varrho$, $p \to p$, $q^i \to Q^i$ and $\tau^{ij} \to -\Sigma^{ij}$
- new degree of freedom: mass density
- new equation: continuity

Back to Carrollian: $2d + 2$ equations

$$\begin{cases} 0 = \frac{k}{\Omega}\nabla_\mu T^\mu{}_0 = \frac{1}{k^2}\mathcal{F} + \mathcal{E} + \mathrm{O}\left(k^2\right) \\ 0 = \nabla_\mu T^{\mu i} = \frac{1}{k^2}\mathcal{H}^i + \mathcal{G}^i + \mathrm{O}\left(k^2\right) \end{cases}$$

Carrollian-covariant equations

Structure

Carrollian-covariant time and space derivatives acting on Carrollian momenta

Carrollian momenta

$$\begin{cases} \frac{1}{\Omega^2} T_{00} = e_{\mathrm{e}} + \mathrm{O}\left(k^2\right) \\ \frac{1}{k\Omega} T^i_{\ 0} = -\frac{\Pi^i}{k^2} - \Upsilon^i + \mathrm{O}\left(k^2\right) \\ T^{ij} = -\frac{\Sigma^{ij}}{k^2} + \Pi^{ij} + \mathrm{O}\left(k^2\right) \end{cases}$$

with

$$\begin{cases} e_{\mathrm{e}} = \varepsilon + 2\beta_i Q^i - \beta_i \beta_j \Sigma^{ij} \\ \Pi^i = Q^i - \beta_j \Sigma^{ij} \\ \Upsilon^i = \pi^i - \beta_k \Xi^{ki} + \beta^i \left(\varepsilon + p + \beta_k Q^k\right) + \frac{\boldsymbol{\beta}^2}{2} Q^i \\ \Pi^{ij} = p a^{ij} - \Xi^{ij} + Q^i \beta^j + \beta^i Q^j \end{cases}$$

Explicit expressions [for Carrollian connection and curvature see Ciambelli et al. '18]

$$
\begin{cases}
\mathcal{E} = -\left(\frac{1}{\Omega}\partial_t + \theta\right) e_{\mathrm{e}} - \left(\hat{\nabla}_i + 2\varphi_i\right)\Pi^i - \Pi^{ij}\left(\xi_{ij} + \frac{1}{d}\theta a_{ij}\right) \\
\mathcal{F} = \Sigma^{ij}\xi_{ij} + \frac{1}{d}\Sigma^i{}_i\theta \quad \text{“constraint”} \\
\mathcal{G}^j = \left(\frac{1}{\Omega}\partial_t + \theta\right)\Upsilon^j + \left(\hat{\nabla}_i + \varphi_i\right)\Pi^{ij} + \varphi^j e_{\mathrm{e}} + 2\Pi^i\varpi_i{}^j \\
\mathcal{H}^j = -\left(\hat{\nabla}_i + \varphi_i\right)\Sigma^{ij} + \left(\frac{1}{\Omega}\partial_t + \theta\right)\Pi^j \quad \text{“continuity”}
\end{cases}
$$

Applications: holographic fluids, Cotton tensor in 3 dim, membrane paradigm [Ciambelli et al. '18 & '20; Campoleoni et al. '19; Donnay, Marteau '19]

Comment on an apparent disagreement [cf. Niels Obers' talk]

- Minkowski metric $\mathrm{d}s^2 = -k^2\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$
- $T^i_{\ 0} = -T_i^{\ 0} \quad (x^0 = kt)$
- $T^i_{\ t} = -k^2 T_i^{\ t} \xrightarrow[k\to 0]{} 0 \quad \text{if} \quad T_i^{\ t} = \mathrm{O}\left(k^\alpha\right) \quad \alpha > -2$
- $T_i^{\ t} = -1/k^2\, T^i_{\ t} \xrightarrow[k\to\infty]{} 0 \quad \text{if} \quad T^i_{\ t} = \mathrm{O}\left(k^\alpha\right) \quad \alpha < 2$

Quasi-alternative method [Ciambelli, Marteau '19]

Carrollian-invariant action and momenta

$$\begin{cases} \Pi^{ij} = \frac{2}{\sqrt{a}\Omega}\frac{\delta S}{\delta a_{ij}} \\ \Pi^{i} = \frac{1}{\sqrt{a}\Omega}\frac{\delta S}{\delta b_{i}} \neq 0 \\ e_{\mathrm{e}} = -\frac{1}{\sqrt{a}}\left(\frac{\delta S}{\delta \Omega} + \frac{b_i}{\Omega}\frac{\delta S}{\delta b_i}\right) \end{cases}$$

[again vs. talk by Niels Obers]

Carrollian-diffeo invariance $\xi = \xi^t(t,\mathbf{x})\partial_t + \xi^i(\mathbf{x})\partial_i$

$$\begin{cases} 0 = -\left(\frac{1}{\Omega}\partial_t + \theta\right)e_{\mathrm{e}} - \left(\hat{\nabla}_i + 2\varphi_i\right)\Pi^i - \Pi^{ij}\left(\xi_{ij} + \frac{1}{d}\theta a_{ij}\right) \\ 0 = \Sigma^{ij}\xi_{ij} + \frac{1}{d}\Sigma^i{}_i\theta \\ 0 = \left(\frac{1}{\Omega}\partial_t + \theta\right)\Upsilon_j + \left(\hat{\nabla}_i + \varphi_i\right)\Pi^i{}_j + \varphi_j e_{\mathrm{e}} + 2\Pi^i\varpi_{ij} \\ 0 = -\left(\hat{\nabla}_i + \varphi_i\right)\Sigma^i{}_j + \left(\frac{1}{\Omega}\partial_t + \theta\right)\Pi_j \end{cases}$$

some degrees of freedom are missing $\rightarrow$ need $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{\Omega}$

Ricci-flat in incomplete Newman–Unti gauge

First hint in $n = 4$ $\lim_{k\to 0} \mathrm{d}s^2_{\text{Einstein}}$ [Ciambelli et al. '18]

$\mathrm{d}s^2_{\text{Ricci-flat}}$ described in terms of *Carrollian boundary data*

- Carrollian geometry (6)
  - degenerate metric (3)
  - Ehresmann connection (3)
- Carrollian fluid (5)
  - energy (1)
  - momenta – heat current (2) and stress tensor (2)
- Carrollian-fluid "velocity" (2) – hydro-frame freedom

Full solution space [Brussels school]

infinite number of further Carrollian data obeying Carrollian dynamics – at every $\mathrm{O}\left(1/r^n\right)$

The holographic Carrollian fluid

Transport – in red fed by the boundary Cotton

- $\varepsilon = 2p \to \varepsilon_{(-1)} k^2 + \varepsilon + \frac{\varepsilon_{(1)}}{k^2} + \frac{\varepsilon_{(2)}}{k^4}$ $\quad \varepsilon \propto$ Bondi mass
- $\frac{q^i}{k^2} \to \pi^i + \frac{Q^i}{k^2} + \frac{\zeta^i}{k^4}$ $\quad \pi^i \propto$ angular momentum aspect
- $\tau^{ij} \to -k^2 E^{ij} - \Xi^{ij} - \frac{\Sigma^{ij}}{k^2} - \frac{\zeta^{ij}}{k^4}$

Non-trivial fluid equations (2)

$$\frac{1}{\Omega}\hat{\mathscr{D}}_t\varepsilon + \hat{\mathscr{D}}_i \frac{*\chi^i}{8\pi G} = \overbrace{\frac{1}{16\pi G}\left(\hat{\mathscr{D}}_i\hat{\mathscr{D}}_j\hat{\mathscr{N}}^{ij} + \mathscr{C}^{ij}\hat{\mathscr{D}}_i\hat{\mathscr{R}}_j + \frac{1}{2}\mathscr{C}_{ij}\frac{1}{\Omega}\hat{\mathscr{D}}_t\hat{\mathscr{N}}^{ij}\right)}^{\text{bry. source – bulk radiation}}$$

$$\frac{1}{\Omega}\hat{\mathscr{D}}_t\left(\pi^i - \underbrace{\frac{*\psi^i}{8\pi G}}_{\text{magnetic}}\right) + \frac{1}{2}\hat{\mathscr{D}}^i\left(\varepsilon + \underbrace{\frac{\hat{\eta}^i{}_j c}{8\pi G}}_{\text{magnetic}}\right) = \underbrace{\text{(shear, news)}}_{\textit{source}}$$

(in $2+1$ dimensions the source is the anomaly in flat *and* AdS)

Ricci-flat spacetimes up to $O(1/r^2)$

$$ds^2_{\text{Ricci-flat}} = 2\boldsymbol{\mu}\left(\mathrm{d}r + r\varphi_a\boldsymbol{\mu}^a - r\frac{\theta}{2}\boldsymbol{\mu} + *\boldsymbol{\mu}^b\hat{\mathscr{D}}_b * \varpi - \frac{1}{2}\boldsymbol{\mu}^a\hat{\mathscr{D}}_b\mathscr{C}^b{}_a\right)$$
$$+\left(\rho^2 + \frac{\mathscr{C}_{cd}\mathscr{C}^{cd}}{8}\right)\mathrm{d}\ell^2 + \mathscr{C}_{ab}\left(r\boldsymbol{\mu}^a\boldsymbol{\mu}^b - *\varpi * \boldsymbol{\mu}^a\boldsymbol{\mu}^b\right)$$
$$+\frac{1}{r}\left[\left(8\pi G\varepsilon - \hat{\mathscr{K}}\right)\boldsymbol{\mu}^2 + \frac{32\pi G}{3}\left(\pi_a - \frac{1}{8\pi G} * \psi_a\right)\boldsymbol{\mu}\boldsymbol{\mu}^a\right.$$
$$\left.-\frac{16\pi G}{3}E_{ab}\boldsymbol{\mu}^a\boldsymbol{\mu}^b\right] + O(1/r^2)$$

$$\begin{cases}\mu = \lim_{k\to 0}\frac{\mathbf{u}}{k^2}\\ \rho^2 = r^2 + \star\varpi^2\end{cases}$$

Hints for flat holography

The asymptotically AdS paradigm in 4 dimensions

- on the Riemannian boundary
  - metric $g_{\mu\nu} \to$ source
  - energy–momentum tensor $T^{\mu\nu} \to$ vev
- Dirichlet bry. conds. $\to$ global $SO(3,2)$ on Minkowski

holographic dual theory $\equiv$ CFT on Minkowski

The 4-dim asymptotically flat expectations

- on the Carrollian boundary
  - $\{a_{ij}, b_i, \Omega\} \to$ source – possibly
  - momenta $\left\{E^{ij}, \pi^i, \varepsilon\right\} \to$ vev – possibly
  - $\ldots\infty$ ($\ni \mathscr{C}_{ab}$ except for 3-dim bulk)
- Dirichlet bry. conds. $\to$ global $\text{CCarroll}_3 \equiv \text{BMS}_4$ on $\mathbb{R} \times \mathbb{E}_2$

dual *non-local* field theory on $\mathbb{R} \times \mathbb{E}_2$ invariant under $\text{BMS}_4 \equiv \text{sT} \rtimes SL(2,\mathbb{C})$ – *Chthonian* Carroll CFT

WHAT ABOUT FLAT$_4$/CFT$_2$ *CELESTIAL* HOLOGRAPHY? [HARVARD SCHOOL]

FRAMEWORK

- $\mathbb{S}^2 \equiv$ spatial section of the Carrollian bry.
- 2-dim en.–mom. tensor $\sim \int \mathscr{N}_{ab} \sim \int \partial_t \mathscr{C}_{ab}$

FEATURES

- limited to "$SL(2, \mathbb{C})$" invariance – vs. BMS$_4$
- ignores the *deep* degrees of freedom
- raises questions about *unitarity* and *locality*

OUTPUT

- kinematic book-keeping for *radiation* S-matrix
- very special to $n = 4$ (e.g. no shear in $n = 3$)

Facts & quotable

- Carrollian fluid $\equiv$ formal Carrollian-general-covariant hydrodynamics
  - enforced by Ricci-flat bounbary dynamics
  - applicable in the membrane-paradigm [Donnay, Marteau '19]
- Flat bulk is mapped onto Carrollian boundary dynamics
  - supports a *Carrollian-fluid/flat-gravity* sector
  - requires infinite sets of *deep* boundary data
  - suggests flat-holographic duals are *non-local* field theories

Worth investigating

- find other applications of Carrollian fluids [cf. Dutch school]
- pursue the quest of BMS-invariant field theories [Le Bellac, Lévy-Leblond '67 & '73; Souriau '85; Duval et al. '14; Bagchi et al. '20; Henneaux, Salgado–Rebolledo '79 & '21]
- circumscribe the role/validity of celestial CFT [Donnay et al. '22]
  - group theory and representation aspects [cf. Glenn Barnich' talk]
  - Carrollian momenta and reconstruction properties
  - better understand the role of boundary Cotton

Carrollian geometry [Duval et al. '14; Bekaert et al. '16; Ciambelli et al. '19]

Basic ingredients in $d+1$ dimensions

- degenerate metric: $\mathrm{d}\ell^2 = a_{ij}(t,\mathbf{x})\mathrm{d}x^i\mathrm{d}x^j \quad i,j = 1,\ldots,d$
- Ehresmann connection: $\boldsymbol{e} = \Omega \mathrm{d}t - b_i \mathrm{d}x^i$

General covariance

Carrollian diffeomorphisms: $t' = t'(t,\mathbf{x}) \quad \mathbf{x}' = \mathbf{x}'(\mathbf{x})$

Example: zero-c limit of Minkowski spacetime [Lévy-Leblond '65]

- $\mathrm{d}\ell^2 = \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \boldsymbol{e} = \mathrm{d}t$
- isometries: Carroll group $\begin{cases} t' = t + B_i x^i + t_0, \\ x'^k = R^k_i x^i + x^k_0 \end{cases}$

Property

$\mathrm{CCarrroll}_{d+1} \equiv \mathrm{BMS}_{d+2}$

Properties

Bulk ASG matches the boundary invariances

- Weyl $\omega(t, \mathbf{x})$
- sT $f(t, \mathbf{x}) \rtimes$ sR $Y^i(\mathbf{x}) \equiv$ Carrollian diffeos
- Carrollian hydrodynamic-frame transformations

Dirichlet ($\mathrm{d}\ell^2 \simeq S^2$ & no Ehresmann) $\to$ $\mathrm{BMS}_4 \equiv \mathrm{CCarroll}_3$

Further comments

- Shear $\mathscr{C}_{ij} \to$ independent $\to$ news $\hat{\mathscr{N}}_{ij}$
- A *remarkable* "self-duality" condition $\to$ resummation – no shear, pure fluid, Carrollian momenta $\equiv$ Carrollian Cotton
  $\mathrm{d}s^2_{\text{res. Ricci-flat}} = \lim_{k\to 0} \mathrm{d}s^2_{\text{res. Einstein}}$

A *miraculous* "self-duality" condition [Ciambelli et al. '18]

$6+1$ independent boundary data

- Carrollian fluid momenta $\equiv$ Carrollian Cotton tensor
- "velocity" $\boldsymbol{\mu} = -\boldsymbol{e}$ (Ehresmann connection)
- zero shear & other Carrollian data frozen

Algebraically special – flat limit of $ds^2_{\text{resummed AdS}}$

$$\begin{aligned} ds^2_{\text{resummed flat}} &= 2\boldsymbol{\mu}\left(\mathrm{d}r + r\boldsymbol{\alpha} + \frac{r\theta\Omega}{2}\mathrm{d}t\right) + r^2\mathrm{d}\ell^2 \\ &\quad + \boldsymbol{s} + \frac{\boldsymbol{\mu}^2}{\rho^2}\left(8\pi G\varepsilon r + c * \varpi\right) \end{aligned}$$

$\rho^2 = r^2 + *\varpi^2$

General covariance and Weyl invariance

Fluid equations covariant – diffeomorphism invariance

Diffeomorphisms are generated by vector fields ($i, j = 1, \ldots, d$)

$$\xi = f\partial_t + Y^i\partial_i$$

$f(t, \mathbf{x})$ and $Y^i(t, \mathbf{x})$ $d + 1$ functions of time and space

$$\delta_\xi = -\mathscr{L}_\xi$$

Conformal (Weyl-covariant) fluids: fluid equations invariant under arbitrary rescaling of the metric

$$\delta_\omega g_{\mu\nu} = -2\omega g_{\mu\nu} \quad \delta_\omega \boldsymbol{u}^\mu = \omega \boldsymbol{u}^\mu$$

$\omega(t, \mathbf{x})$ arbitrary function of time and space

$$\delta_\omega = w\omega$$

The hydrodynamic-frame invariance

Landau–Lifshitz's following 1940 Eckart's statements

[Theoretical Physics vol. 6 §136]

u^μ is not physical/measurable – a book-keeping device

Translation: gauge invariance

Arbitrary *local* Lorentz transformations of u^μ can be compensated by appropriate modifications of $T, \varepsilon, p, q^\mu, \tau^{\mu\nu}$ such that $T^{\mu\nu}$ *and* the entropy current S^μ remain invariant

Note: These are *not* Lorentz isometries (generally absent) but tangent-space *local* transformations generated by Z^i (d boosts), S_{ij} antisymmetric ($^{d(d-1)}/_2$ rotations)

In summary

Conformal-fluid symmetries on arbitrary backgrounds

∞-dim generated by $\{\omega(t,\mathbf{x}), f(t,\mathbf{x}), Y^i(t,\mathbf{x}), Z^i(t,\mathbf{x}), S_{ij}(t,\mathbf{x})\}$

A GOOD GAUGE IN $d+1$ DIMENSIONS

RIEMANNIAN $\mathscr{M}$ IN ADM/ZERMELO

$$\mathrm{d}s^2 = -\Omega^2 k^2 \mathrm{d}t^2 + a_{ij}\left(\mathrm{d}x^i - w^i \mathrm{d}t\right)\left(\mathrm{d}x^j - w^j \mathrm{d}t\right)$$

with $\Omega = \Omega(t)$

- Galilean diffeos $t' = t'(t), \mathbf{x}' = \mathbf{x}'(t, \mathbf{x})$ (here $x^0 = kt$)

  $J^{\mu}_{\nu}(x) = \frac{\partial x^{\mu\prime}}{\partial x^{\nu}} = \begin{pmatrix} J(t) & 0 \\ \frac{j^i(x)}{c} & J^i_j(x) \end{pmatrix} \to$ reduction of V^{μ}

  - ΩV^0 is a Galilean scalar
  - V_i is a Galilean form

- $\mathscr{M} \underset{k\to\infty}{\longrightarrow} \mathbb{R} \times \mathscr{S}$ Galilean geometry with inverse metric $a^{ij}\partial_i\partial_j$ & time arrow $\mathbf{e} = \frac{1}{\Omega}\left(\partial_t + w^i\partial_i\right)$

The Galilean limit of fluids

Kinematics

$$\mathbf{u} = \gamma\left(-\partial_t + v^i \partial_i\right)$$

with $u^\mu u_\mu = -k^2$

Transport

- $\varepsilon \to e\varrho + k^2 \varrho$ and $p \to p$
- $q^i \to Q^i$ and $\tau^{ij} \to -\Sigma^{ij}$

scalings suggested by out-of-equilibrium thermodynamics

Galilean-covariant equations

Structure of the $d + 2$ equations

$$\begin{cases} 0 = k\Omega\nabla_\mu T^{\mu 0} = k^2\mathcal{C} + \mathcal{Q} + \mathrm{O}\left(1/k^2\right) \\ 0 = \nabla_\mu T^\mu{}_i = \mathcal{M}_i + \mathrm{O}\left(1/k^2\right) \end{cases}$$

Galilean momenta

$$\begin{cases} \Omega^2 T^{00} = k^2\varrho + \Pi + \mathrm{O}\left(1/k^2\right) \\ k\Omega T_i{}^0 = k^2 P_i + \Pi_i + \mathrm{O}\left(1/k^2\right) \\ T_{ij} = \Pi_{ij} + \mathrm{O}\left(1/k^2\right) \end{cases}$$

with

$$\begin{cases} \Pi = \varrho\left(e + \frac{1}{2}\left(\frac{\mathbf{v}-\mathbf{w}}{\Omega}\right)^2\right) \\ P_i = \varrho\frac{v_i - w_i}{\Omega} \\ \Pi_i = Q_i - \frac{v^j - w^j}{\Omega} + \varrho\frac{v_i - w_i}{\Omega}\left(h + \frac{1}{2}\left(\frac{\mathbf{v}-\mathbf{w}}{\Omega}\right)^2\right)\Sigma_{ji} \\ \Pi_{ij} = p a_{ij} - \Sigma_{ij} + \varrho\frac{(v_i - w_i)(v_j - w_j)}{\Omega^2} \end{cases}$$

Explicit expressions

$$\begin{cases} \mathcal{C} = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}\varrho}{\mathrm{d}t} + \hat{\nabla}_j P^j \\ \mathcal{Q} = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}\Pi}{\mathrm{d}t} + \hat{\nabla}_i \Pi^i + \Pi^{ij}\gamma_{ij} \\ \mathcal{M}_i = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}P_i}{\mathrm{d}t} + \hat{\nabla}_j \Pi_i^{\ j} \end{cases}$$

Remark – before considering the Galilean limit

- $J^\mu = \varrho_0 u^\mu + j^\mu$
- $\nabla_\mu J^\mu = 0$
- $p + \varepsilon = T\sigma + \mu_0 \varrho_0 \quad \mathrm{d}\varepsilon = T\mathrm{d}\sigma + \mu_0 \mathrm{d}\varrho_0$
- $\mu_0 = \mu + k^2 \quad \varepsilon = \left(e + k^2\right)\varrho_0$

Remark – in the Galilean limit

- $\varrho_0 = \varrho + \mathrm{O}\left(1/k^2\right)$
- $J_i = P_i + \frac{i_i}{k^2} + \mathrm{O}\left(1/k^4\right)$

Quasi-alternative method

Galilean-invariant action and momenta

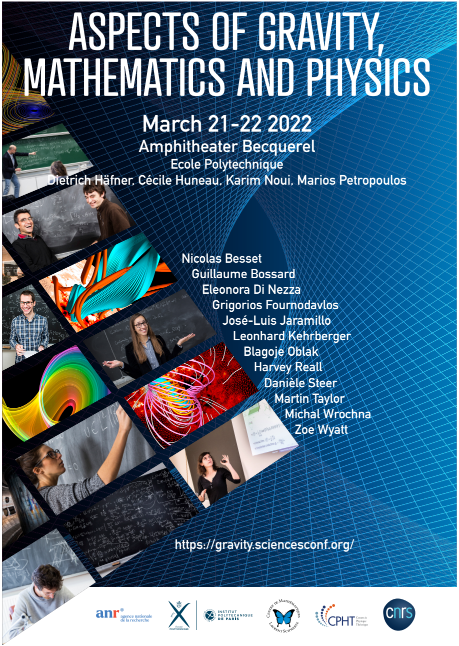
$$\begin{cases} \Pi_{ij} = -\frac{2}{\sqrt{a}\Omega}\frac{\delta S}{\delta a^{ij}} \\ P_i = -\frac{1}{\sqrt{a}\Omega}\frac{\delta S}{\delta \frac{w^i}{\Omega}} \\ \Pi = -\frac{1}{\sqrt{a}}\left(\frac{\delta S}{\delta \Omega} - \frac{w^i}{\Omega^2}\frac{\delta S}{\delta \frac{w^i}{\Omega}}\right) \end{cases}$$

Galilean-diffeomorphism invariance

$$\begin{cases} \mathcal{C} = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}\varrho}{\mathrm{d}t} + \hat{\nabla}_j P^j \\ \mathcal{Q} = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}\Pi}{\mathrm{d}t} + \hat{\nabla}_i \Pi^i + \Pi^{ij}\gamma_{ij} \\ \mathcal{M}_i = \frac{1}{\Omega}\frac{\tilde{\mathrm{D}}P_i}{\mathrm{d}t} + \hat{\nabla}_j \Pi_i^{\ j} \end{cases}$$

Missing terms/equations

Further degrees of freedom are needed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