Asymptotic symmetries in Carrollian theories of gravity and Carrollian black holes

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Introduction

Carroll symmetry has been used in very different physical contexts (as it is clear from this conference!)

In gravity → Gravitational theories with a "Carrollian structure"

1) Described in first order formalism:

- Hartong (2015)
- Bergshoeff, Gomis, Rollier, Rosseel, ter Veldhuis (2017)

→ Local Carroll symmetry

2) Described in Hamiltonian formalism:

- "Strong gravity" (Isham 1976) or "zero-signature limit of GR" (Teitelboim 1978) ⟶ Electric Carroll gravity
- Magnetic Carrol gravity (Henneaux, Salgado-Rebolledo 2021)

The algebra of constraints is simpler than in General Relativity
→ field-independent → Hamiltonian constraints form an abelian subalgebra

It could be an interesting framework to explore certain aspects of quantum gravity

In magnetic Carroll gravity: "Carrollian black holes" $\longrightarrow$ Thermal regular solutions in the context of Carrollian geometry $\rightarrow$ Possess a non-trivial entropy

A first step: Analysis of the asymptotic symmetries (with and without cosmological constant)

- Electric theory $\longrightarrow$ "Unwanted features"

- Magnetic theory $\longrightarrow$ Very rich asymptotic structure

Asymptotic symmetries: Summary of the results

	$\Lambda = 0$		$\Lambda < 0$
	Regge-Teitelboim parity conditions	Henneaux-Troessaert parity conditions	No parity conditions
General Relativity	Poincaré	BMS_4	$AdS_4 \simeq so\,(2,3)$
Magnetic Carroll gravity	Carroll algebra	BMS-Carroll algebra	3D Conformal Carroll algebra (BMS_4)
Electric Carroll gravity	Spatial rotations $\inplus$ spatial translations	Spatial rotations $\inplus$ parity odd supertranslations	$so(1,4)$ (Euclidean)

Boosts can be included in the electric theory with $\Lambda = 0$ when "extended Henneaux-Troessaert" parity conditions are considered (Oscar's talk)

Asymptotic symmetries in electric Carroll gravity

Electric Carroll gravity

The electric contraction can be interpreted as a strong coupling limit of Einstein gravity **[Isham (1976)]**, or as a gravitational theory in a "zero-signature limit" **[Teitelboim (1978)]**

$$\mathcal{H}^E = \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) + 2\sqrt{g}\Lambda \qquad , \qquad \mathcal{H}_i^E = -2\pi_{i|j}^{\ j}$$

The constraints $\mathcal{H}^E$ and $\mathcal{H}_i^E$ obey the algebra:

$$\begin{aligned}
\left\{\mathcal{H}^E(x), \mathcal{H}^E(x')\right\} &= 0 \\
\left\{\mathcal{H}^E(x), \mathcal{H}_i^E(x')\right\} &= \mathcal{H}^E(x)\,\delta_{,i}(x,x') \\
\left\{\mathcal{H}_i^E(x), \mathcal{H}_j^E(x')\right\} &= \mathcal{H}_i^E(x')\,\delta_{,j}(x,x') + \mathcal{H}_j^E(x)\,\delta_{,i}(x,x')
\end{aligned}$$

A covariant formulation in **[Henneaux (1979)]**

Asymptotic symmetries in Electric Carroll gravity with $\Lambda = 0$

The generator of gauge symmetries takes the form

$$G\left[\xi, \xi^i\right] = \int d^3x \left(\xi\, \mathcal{H}^E + \xi^i\, \mathcal{H}_i^E\right) + Q_E$$

Here

$$\mathcal{H}^E = \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right), \qquad \mathcal{H}_i^E = -2\pi_{i|j}^{\ j}$$

with

$$\delta Q_E = \oint d^2 s_l \left(2\xi_k \delta\pi^{kl} + \left(2\xi^k \pi^{jl} - \xi^l \pi^{jk}\right)\delta g_{jk}\right)$$

- The lapse function does not appear in the boundary term of the Hamiltonian!

The transformation laws of the canonical variables are given by

$$\delta g_{ij} = \frac{2\xi}{\sqrt{g}}\left(\pi_{ij} - \frac{1}{2} g_{ij}\pi\right) + \xi_{i|j} + \xi_{j|i}$$

$$\delta\pi^{ij} = \frac{\xi}{2\sqrt{g}} g^{ij}\left(\pi^{kl}\pi_{kl} - \frac{1}{2}\pi^2\right) - \frac{2\xi}{\sqrt{g}}\left(\pi^i{}_l\pi^{lj} - \frac{1}{2}\pi\pi^{ij}\right) + \mathcal{L}_\xi \pi^{ij}$$

Fall-off of the canonical variables

We consider deviations with respect to the Carrollian ground state characterized by

$$\bar{g}_{ij} dx^i dx^j = dr^2 + r^2 \gamma_{AB} dx^A dx^B \,, \qquad \bar{\pi}^{ij} = 0$$

with $A, B = 1, 2$, and where γ_{AB} denotes the metric of the round 2-sphere.

The proposed fall-off is:

$$g_{rr} = 1 + \frac{f_{rr}}{r} + \frac{f_{rr}^{(-2)}}{r^2} + \mathcal{O}\left(r^{-3}\right)$$
$$g_{rA} = \frac{f_{rA}^{(-1)}}{r} + \mathcal{O}\left(r^{-2}\right)$$
$$g_{AB} = r^2 \gamma_{AB} + r\, f_{AB} + f_{AB}^{(0)} + \mathcal{O}\left(r^{-1}\right)$$

$$\pi^{rr} = p^{rr} + \mathcal{O}\left(r^{-1}\right)$$
$$\pi^{rA} = \frac{p^{rA}}{r} + \frac{p^{rA}_{(-2)}}{r^2} + \mathcal{O}\left(r^{-3}\right)$$
$$\pi^{AB} = \frac{p^{AB}}{r^2} + \mathcal{O}\left(r^{-3}\right)$$

The asymptotic form of the parameters that preserve the fall-off is:

$$\xi = r\,b + f + \mathcal{O}\left(r^{-1}\right)\,,\qquad \xi^{r} = W + \mathcal{O}\left(r^{-1}\right)$$
$$\xi^{A} = Y^{A} + \frac{1}{r}\left(\frac{2b}{\sqrt{\gamma}}p^{rA} + D^{A}W\right) + \mathcal{O}\left(r^{-2}\right)$$

where $Y^{A} = \frac{\epsilon^{AB}}{\sqrt{\gamma}}\partial_{B}\left(\vec{\omega}\cdot\hat{r}\right)$.

For large values of r, the symplectic term is logarithmically divergent

$$\int dt d^{3}x\,\pi^{ij}\,\dot{g}_{ij} \underset{r\to\infty}{=} \log\left(r\right)\int dt \oint d^{2}x\left(p^{rr}\dot{f}_{rr} + p^{AB}\dot{f}_{AB}\right) + \mathcal{O}\left(r^{-1}\right)$$

The divergence can be removed by imposing appropriate parity conditions on the canonical variables.

Regge-Teitelboim parity conditions

Under the antipodal map

$$\theta \to -\theta + \pi \qquad , \qquad \phi \to \phi + \pi$$

the Regge-Teitelboim parity conditions are given by

$$f_{rr},\, f_{\theta\theta},\, f_{\phi\phi},\, p^{\theta\phi},\, p^{r\theta} \qquad \text{(parity even)}$$

$$f_{\theta\phi},\, p^{rr},\, p^{\theta\theta},\, p^{\phi\phi},\, p^{r\phi} \qquad \text{(parity odd)}$$

The symplectic term is finite

From the parity conditions:

$$b = b_{\text{odd}}\left(\theta,\phi\right)\ , \qquad W\left(\theta,\phi\right) = \vec{\alpha}\cdot\hat{r} + W_{\text{even}}\left(\theta,\phi\right)$$

The charge takes the form

$$Q_E = \vec{\omega}\cdot\vec{J} + \vec{\alpha}\cdot\vec{P}$$

with

$$J_I = 2\oint d^2x\,\sqrt{\gamma}\,\hat{r}_I\epsilon_{AB}D^A p^{rB}_{(-2)}\ , \qquad P_I = 2\oint d^2x\,\hat{r}_I\left(p^{rr} - 2\tilde{p}\right)$$

which obey

$$\{J_I, J_J\} = -\epsilon_{IJK}J_K\ , \qquad \{P_I, J_J\} = -\epsilon_{IJK}P_K\ , \qquad \{P_I, P_J\} = 0$$

The asymptotic symmetry algebra of electric Carroll gravity with $\Lambda = 0$ and Regge-Teitelboim parity conditions is the semi-direct sum of spatial rotations with spatial translations

There is no notion of energy in this theory!

Henneaux-Troessaert parity conditions

To implement the Henneaux-Troessaert parity conditions it is useful to introduce the following variables

$$\bar{\lambda} = \frac{1}{2} f_{rr}, \quad \bar{k}_{AB} = \frac{1}{2} f_{AB} + \frac{1}{2} f_{rr} \gamma_{AB}, \quad \bar{k} = \frac{1}{2} \tilde{f} + f_{rr}, \quad \bar{p} = 2 \left(p^{rr} - p^{AB} \gamma_{AB} \right)$$

The Henneaux-Troessaert parity conditions are given by

$$\begin{aligned} &\bar{\lambda},\, p^{r\phi},\, p^{\theta\theta},\, p^{\phi\phi},\, \bar{k}_{\theta\phi} \qquad && \text{(parity even)} \\ &\bar{p},\, p^{r\theta},\, p^{\theta\phi},\, \bar{k}_{\theta\theta},\, \bar{k}_{\phi\phi} \qquad && \text{(parity odd)} \end{aligned}$$

The symplectic term takes the form

$$\int dt d^3x\, \pi^{ij} \dot{g}_{ij} \underset{r \to \infty}{=} \log(r) \int dt \oint d^2x \left(\bar{p} \dot{\bar{\lambda}} + 2 p^{AB} \dot{\bar{k}}_{AB} \right) + \mathcal{O}\left(r^{-1}\right)$$

expression that vanishes by virtue of the parity conditions.

In this case we have

$$\begin{aligned} b\,(\theta,\phi) \qquad & \text{(parity odd)} \\ W\,(\theta,\phi) \qquad & \text{(parity odd)} \end{aligned}$$

The charge takes the form

$$Q_E = \vec{\omega}\cdot\vec{J} + \oint d^2x\,\sqrt{\gamma}\,W\,(\theta,\phi)\;\mathcal{P}\,(\theta,\phi)$$

with

$$J_I = 2\oint d^2x\,\sqrt{\gamma}\epsilon_{AB}\hat{r}_I\,D^A\left(p^{rB}_{(-2)} - 2\bar{\lambda}p^{rB}\right)\,, \qquad \mathcal{P}\,(\theta,\phi) = \frac{\bar{p}}{\sqrt{\gamma}}$$

Their algebra is given by

$$\{J_I, J_J\} = -\epsilon_{IJK}J_K\,,\;\; \{\mathcal{P}\,(\theta,\phi)\,, J_I\} = \hat{Y}_I^A\partial_A\mathcal{P}\,(\theta,\phi)\,,\;\; \left\{\mathcal{P}\,(\theta,\phi)\,, \mathcal{P}\left(\theta',\phi'\right)\right\} = 0$$

where $\hat{Y}_I^A := \frac{\epsilon^{AB}}{\sqrt{\gamma}}\partial_B\hat{r}_I$.

The asymptotic symmetry algebra of electric Carroll gravity with $\Lambda = 0$ and Henneaux-Troessaert parity conditions is the semi-direct sum of spatial rotations with parity odd supertranslations $\longrightarrow$ Infinite dimensional

Asymptotic symmetries in Electric Carroll gravity with $\Lambda < 0$

When a negative cosmological is present, the solution obtained from a direct Carrollian limit of AdS spacetime, given by

$$\bar{g}_{ij} dx^i dx^j = \frac{dr^2}{\left(\frac{r^2}{l^2} + 1\right)} + r^2 \gamma_{AB} dx^A dx^B \,, \qquad \bar{\pi}^{ij} = 0$$

$$\bar{N} = \sqrt{\frac{r^2}{l^2} + 1} \,, \qquad \bar{N}^i = 0$$

is not a solution of the Hamiltonian constraint

$$\mathcal{H}^E = \frac{1}{\sqrt{g}} \left(\pi^{ij} \pi_{ij} - \frac{1}{2} \pi^2 \right) + 2\sqrt{g}\Lambda$$

[Hansen, Obers, Oling, Sogaard (2021)]

As a consequence, it does not seem to be possible to construct a consistent set of asymptotic conditions for this theory.

Possible solution: To consider the electric Carrollian theory obtained from *Euclidean* General Relativity

$$\mathcal{H}^{E}_{\text{Euc}} = -\frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^{2}\right) + 2\sqrt{g}\Lambda$$

Alternative ground state:

$$\bar{g}_{ij}dx^{i}dx^{j} = \frac{dr^{2}}{\left(\frac{r^{2}}{l^{2}}+1\right)} + r^{2}\gamma_{AB}dx^{A}dx^{B}\,, \qquad \bar{\pi}^{ij} = \frac{2}{l}\sqrt{\bar{g}}\bar{g}^{ij}$$

It is possible to construct a set of asymptotic conditions in terms of deviations with respect to this background configuration.

Asymptotic symmetry algebra: $so(1,4) \rightarrow$ non-Carrollian.

Furthermore, the space of spherically symmetric solutions of this theory is degenerate. $\rightarrow$ The theory as “unwanted properties.”

Asymptotic symmetries in magnetic Carroll gravity

Magnetic Carroll gravity

$$\mathcal{H}^M = -\sqrt{g}\,(R - 2\Lambda) \qquad , \qquad \mathcal{H}_i^M = -2\pi_{i|j}^{\;j}$$

The constraints $\mathcal{H}^M$ and $\mathcal{H}_i^M$ obey the algebra:

$$\left\{\mathcal{H}^M(x), \mathcal{H}^M(x')\right\} = 0$$
$$\left\{\mathcal{H}^M(x), \mathcal{H}_i^M(x')\right\} = \mathcal{H}^M(x)\,\delta_{,i}(x, x')$$
$$\left\{\mathcal{H}_i^M(x), \mathcal{H}_j^M(x')\right\} = \mathcal{H}_i^M(x')\,\delta_{,j}(x, x') + \mathcal{H}_j^M(x)\,\delta_{,i}(x, x')$$

[**Henneaux, Salgado-Rebolledo (2021)**]

A covariant formulation in [**Hansen, Obers, Oling, Sogaard (2021)**]

Asymptotic symmetries in Magnetic Carroll gravity with $\Lambda = 0$

The generator of gauge symmetries takes the form

$$G\left[\xi,\xi^{i}\right]=\int d^{3}x\left(\xi\,\mathcal{H}^{M}+\xi^{i}\,\mathcal{H}_{i}^{M}\right)+Q_{M}$$

where

$$\delta Q_{M}=\oint d^{2}s_{l}\left[G^{ijkl}\left(\xi\delta g_{ij|k}-\xi_{|k}\delta g_{ij}\right)+2\xi_{k}\delta\pi^{kl}+\left(2\xi^{k}\pi^{jl}-\xi^{l}\pi^{jk}\right)\delta g_{jk}\right]$$

with $G^{ijkl}=\frac{1}{2}\sqrt{g}\left(g^{ik}g^{jl}+g^{il}g^{jk}-2g^{ij}g^{kl}\right)$

- The same as in General Relativity.

The transformation laws for the canonical variables are given by

$$\delta g_{ij}=\xi_{i|j}+\xi_{j|i}$$

$$\delta\pi^{ij}=-\xi\sqrt{g}\left(R^{ij}-\frac{1}{2}g^{ij}R\right)+\sqrt{g}\left(\xi^{|i|j}-g^{ij}\xi_{|k}^{|k}\right)+\mathcal{L}_{\xi}\pi^{ij}$$

$$
\begin{aligned}
g_{rr} &= 1 + \frac{f_{rr}}{r} + \frac{f_{rr}^{(-2)}}{r^2} + \mathcal{O}\left(r^{-3}\right) \\
g_{rA} &= \frac{f_{rA}^{(-1)}}{r} + \mathcal{O}\left(r^{-2}\right) \\
g_{AB} &= r^2 \gamma_{AB} + r\, f_{AB} + f_{AB}^{(0)} + \mathcal{O}\left(r^{-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi^{rr} &= p^{rr} + \mathcal{O}\left(r^{-1}\right) \\
\pi^{rA} &= \frac{p^{rA}}{r} + \frac{p^{rA}_{(-2)}}{r^2} + \mathcal{O}\left(r^{-3}\right) \\
\pi^{AB} &= \frac{p^{AB}}{r^2} + \mathcal{O}\left(r^{-3}\right)
\end{aligned}
$$

The fall-off is preserved by the following gauge parameters:

$$
\begin{aligned}
\xi &= r\, b + f\left(\theta, \phi\right) + \mathcal{O}\left(r^{-1}\right) \\
\xi^r &= W\left(\theta, \phi\right) + \mathcal{O}\left(r^{-1}\right) \\
\xi^A &= Y^A + \frac{\partial^A W\left(\theta, \phi\right)}{r} + \mathcal{O}\left(r^{-2}\right)
\end{aligned}
$$

with

$$
b = \vec{\beta} \cdot \hat{r}\,, \qquad Y^A = \frac{\epsilon^{AB}}{\sqrt{\gamma}} \partial_B \left(\vec{\omega} \cdot \hat{r}\right)
$$

Regge-Teitelboim parity conditions

$$W = \vec{\alpha} \cdot \hat{r} + W_{\text{even}}\left(\theta, \phi\right) , \qquad f = T + f_{\text{odd}}\left(\theta, \phi\right)$$

where W_{even} and f_{odd} are pure gauge.

The charge then takes the form

$$Q_M = T\,E + \vec{\omega} \cdot \vec{J} + \vec{\alpha} \cdot \vec{P} + \vec{\beta} \cdot \vec{K}$$

with

$$E = 2 \oint d^2x \, \sqrt{\gamma} f_{rr} \, , \qquad P_I = 2 \oint d^2x \, \hat{r}_I \left(p^{rr} - D_A p^{rA} \right)$$

$$K_I = 2 \oint d^2x \sqrt{\gamma} \hat{r}_I \left(f_{rr}^{(-2)} + D^A f_{rA}^{(-1)} + \tilde{f}^{(0)} \right) , \quad J_I = 2 \oint d^2x \, \sqrt{\gamma} \hat{r}_I \epsilon_{AB} D^A p^{rB}_{(-2)}$$

The generators obey

$$\{P_I, K_J\} = \delta_{IJ} E \, , \qquad \{J_I, J_J\} = -\epsilon_{IJK} J_K$$

$$\{P_I, J_J\} = -\epsilon_{IJK} P_K \, , \qquad \{K_I, J_I\} = -\epsilon_{IJK} K_K$$

The symmetry algebra of magnetic Carroll gravity with $\Lambda = 0$ and Regge-Teitelboim parity conditions is the Carroll algebra

Henneaux-Troessaert parity conditions

To have an integrable charge, the following shift is necessary:

$$f = -\frac{1}{2} b \left(3 f_{rr} + \tilde{f} \right) + T \left(\theta, \phi \right)$$

where the functions $T(\theta,\phi)$ and $W(\theta,\phi)$ have the following parity under the antipodal map

$$\begin{aligned} T \left(\theta, \phi \right) \qquad & \text{(parity even)} \\ W \left(\theta, \phi \right) \qquad & \text{(parity odd)} \end{aligned}$$

The charge takes the form

$$Q_M = \vec{\omega} \cdot \vec{J} + \vec{\beta} \cdot \vec{K} + \oint d^2x \, \sqrt{\gamma} \left[T \left(\theta, \phi \right) \mathcal{T} \left(\theta, \phi \right) + W \left(\theta, \phi \right) \mathcal{P} \left(\theta, \phi \right) \right]$$

with

$$\mathcal{T} \left(\theta, \phi \right) = 4 \bar{\lambda} \, , \qquad \mathcal{P} \left(\theta, \phi \right) = \frac{\bar{p}}{\sqrt{\gamma}}$$

$$J_I = \oint d^2x \, 2 \sqrt{\gamma} \epsilon_{AB} \hat{r}_I \, D^A \left(p^{rB}_{(-2)} - 2 \bar{\lambda} p^{rB} \right) , \quad K_I = \oint d^2x \, 2 \sqrt{\gamma} \, \hat{r}_I \left(k^{(2)} - 3 \bar{\lambda} \bar{k} \right)$$

The generators have the following non-vanishing Poisson brackets

$$\{J_I, J_J\} = -\epsilon_{IJK} J_K , \qquad \{K_I, J_I\} = -\epsilon_{IJK} K_K$$

$$\{\mathcal{P}(\theta,\phi), J_I\} = \hat{Y}_I^A \partial_A \mathcal{P}(\theta,\phi) \ , \qquad \{\mathcal{T}(\theta,\phi), J_I\} = \hat{Y}_I^A \partial_A \mathcal{T}(\theta,\phi)$$

$$\{\mathcal{P}(\theta,\phi), K_I\} = \hat{r}_I (3\mathcal{T} + \Delta\mathcal{T}) + (D_A \hat{r}_I)\left(D^A \mathcal{T}\right)$$

where $\hat{Y}_I^A := \frac{\epsilon^{AB}}{\sqrt{\gamma}} \partial_B \hat{r}_I$.

The symmetry algebra of magnetic Carroll gravity with $\Lambda = 0$ and Henneaux-Troessaert parity conditions is the "BMS-Carroll algebra."

Asymptotic symmetries in Magnetic Carroll gravity with $\Lambda < 0$

We consider the following background solution

$$\bar{g}_{ij}dx^i dx^j = \frac{dr^2}{\left(\frac{r^2}{l^2}+1\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \qquad \bar{\pi}^{ij} = 0$$

$$\bar{N} = \sqrt{\frac{r^2}{l^2}+1} , \qquad \bar{N}^i = 0$$

- Carrollian isometries are given by the Carroll-AdS$_4$ algebra

$$\{J_I, J_J\} = -\epsilon_{IJK} J_K , \qquad \{P_I, J_J\} = -\epsilon_{IJK} P_K , \qquad \{K_I, J_J\} = -\epsilon_{IJK} P_K$$

$$\{P_I, K_J\} = \delta_{IJ} E , \qquad \{P_I, P_J\} = \frac{1}{l^2}\epsilon_{IJK} J_K , \qquad \{P_I, E\} = \frac{1}{l^2} K_I$$

- Defines an homogeneous space that can be obtained from a coset construction. **[Figueroa-O'Farrill, Grassie, Prohazka (2019)]**
- As in three-dimensional gravity, there is an enhancement of these symmetries in the asymptotic symmetry algebra.

Fall-off

The asymptotic behavior of the canonical variables is the following:

$$
\begin{aligned}
g_{rr} &= \frac{l^2}{r^2} - \frac{l^4}{r^4} + \frac{f_{rr}}{r^5} + \mathcal{O}\left(r^{-6}\right) \\
g_{rA} &= \frac{f_{rA}}{r^4} + \mathcal{O}\left(r^{-5}\right) \\
g_{AB} &= r^2\gamma_{AB} + h_{AB} + \frac{f_{AB}}{r} + \mathcal{O}\left(r^{-2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi^{rr} &= \frac{p^{rr}}{r} + \mathcal{O}\left(r^{-2}\right) \\
\pi^{rA} &= -\frac{D_B \tilde{k}^{AB}_{(2)}}{r} + \frac{p^{rA}}{r^2} + \mathcal{O}\left(r^{-3}\right) \\
\pi^{AB} &= \frac{\tilde{k}^{AB}_{(2)}}{r^2} + \frac{k^{AB}_{(4)}}{r^4} + \frac{p^{AB}}{r^5} + \mathcal{O}\left(r^{-6}\right)
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The fall-off is preserved by the following parameters:

$$\xi = \frac{r}{l} T(\theta,\phi) + \frac{l(\Delta+2) T(\theta,\phi)}{4r} + \dots$$
$$\xi^r = -\frac{r}{2} D_A Y^A(\theta,\phi) - \frac{l^2}{4} D_A Y^A(\theta,\phi) \frac{1}{r} + \dots$$
$$\xi^A = Y^A(\theta,\phi) - \frac{l^2}{4r^2} D^A D_B Y^B(\theta,\phi) + \dots$$

where Y^A must obey the conformal Killing equation on the 2-sphere

$$D_A Y_B + D_B Y_A - \gamma_{AB} D_C Y^C = 0$$

- $T(\theta,\phi)$ is an arbitrary function on the sphere

The charge is given by

$$Q_M = \oint d^2x \sqrt{\gamma} \left(T \mathcal{P} + Y^A \mathcal{J}_A \right)$$

where

$$\mathcal{P} := \frac{1}{l^2}\left(3\tilde{f} + \frac{2}{l^2} f_{rr}\right), \qquad \mathcal{J}_A := 2\gamma^{-\frac{1}{2}} \gamma_{AB} p^{rB}$$

The algebra closes according to

$$T_3 = Y_1^A \partial_A T_2 - Y_2^A \partial_A T_1 + \frac{1}{2}\left(T_1 D_A Y_2^A - T_2 D_A Y_1^A\right)$$
$$Y_3^A = Y_1^C \partial_C Y_2^A - Y_2^C \partial_C Y_1^A$$

This is precisely the composition rule of the BMS_4 algebra. **[Bondi, Van der Burg, Metzner (1962)] [Sachs (1962)]**

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- The BMS_4 generators are canonical $\rightarrow$ in contrast with the analysis at null infinity in GR with $\Lambda = 0$.
- Admits Barnich-Troessaert superrotations. **[Barnich, Troessaert (2009)]**
- The asymptotic conditions can be consistently truncated to recover the Carroll-AdS_4 algebra.

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$$Y_3^A = Y_1^C \partial_C Y_2^A - Y_2^C \partial_C Y_1^A$$

This is precisely the composition rule of the BMS_4 algebra. **[Bondi, Van der Burg, Metzner (1962)] [Sachs (1962)]**

- Is infinite-dimensional in contrast to the case in General Relativity.
- The BMS_4 generators are canonical $\rightarrow$ in contrast with the analysis at null infinity in GR with $\Lambda = 0$.
- Admits Barnich-Troessaert superrotations. **[Barnich, Troessaert (2009)]**
- The asymptotic conditions can be consistently truncated to recover the Carroll-AdS_4 algebra.
- Is isomorphic to the three-dimensional conformal Carroll algebra. **[Duval, Gibbons, Horvathy (2014)]**

The algebra closes according to

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- There is full agreement with the expectations coming from holography.

Carrollian black holes

In collaboration with R. Troncoso, *to appear*

Carrollian Schwarzschild solution

The magnetic theory (with $\Lambda = 0$) admits the following solution

$$g_{ij}dx^i dx^j = \frac{dr^2}{\left(1 - \frac{r_+}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \qquad \pi^{ij} = 0$$

The lapse and shift functions are given by

$$N = \sqrt{1 - \frac{r_+}{r}} , \qquad N^i = 0$$

- The canonical variables are the same than in Schwarzschild in General Relativity.

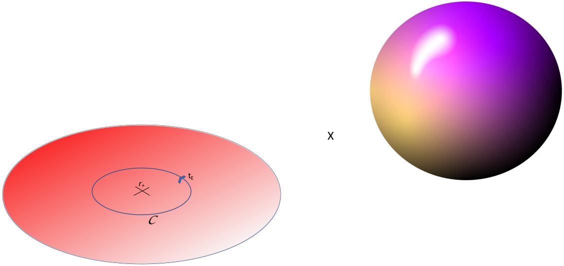
The energy of the solution is

$$E = \frac{r_+}{2G_c}$$

$$\left(\mathcal{H}^M = -\frac{\sqrt{g}}{16\pi G_c}\left(R - 2\Lambda\right) \right)$$

- Can this solution be interpreted as a black hole?

Euclidean black holes



- Topology: Disk $\times$ S^{D-2}.
- In the context of Riemannian geometry: Regularity of the Euclidean metric, or alternatively spin connection with trivial holonomy around the thermal cycle.
- The absence of conical singularities fixes the period of the Euclidean time $\rightarrow$ Hawking temperature.

Carrollian geometry *à la* Cartan

Carrollian geometry can be described in terms of a "clock form," the spatial vielbein

$$\tau = \tau_\mu dx^\mu \,, \qquad e^a = e^a_\mu dx^\mu \,, \qquad a = 1,2,3 \,, \quad \mu = 0,1,2,3$$

and the spin connection

$$\omega^a = \omega^a_\mu dx^\mu \qquad \omega^{ab} = \omega^{ab}_\mu dx^\mu$$

If

$$\omega := \omega^a K_a + \frac{1}{2}\omega^{ab} J_{ab}$$

The condition for the existence of a Carrollian black hole is that the holonomy of ω around the thermal cycle is trivial

$$\mathcal{P}\exp\left(\oint_{\mathcal{C}} \omega\right) = \mathbb{1}$$

For a spherically symmetric solution of the form:

$$ds^2 = \frac{dr^2}{f^2(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad N = N(r)$$

one has

$$\tau = N dt\,, \quad e^1 = \frac{1}{f(r)} dr\,, \quad e^2 = r d\theta, \quad e^3 = r\sin\theta d\phi$$

The spin connection can be expressed in terms of the vielbeins using the torsionless condition (which is compatible with the magnetic theory).
[Bergshoeff, Gomis, Rollier, Rosseel, ter Veldhuis (2017)]

The only relevant component is

$$\omega_t = -f\partial_r N\, K_1$$

The regularity condition becomes

$$\exp\left[i\beta f\partial_r N\, K_1\right] = \mathbb{1}$$

Therefore, the temperature $T = \beta^{-1}$ is fixed as follows:

$$T = \frac{1}{2\pi}\, f\partial_r N|_{r_+}$$

For the Carrollian Schwarzschild solution:

$$T = \frac{1}{4\pi r_+}$$

The entropy can be directly obtained from the boundary terms of the action

$$S = I|_{r_+} = \frac{\pi r_+^2}{G_c} = \frac{A}{4G_c}$$

The first law is automatically fulfilled

$$\delta E = T\delta S$$

- The Carrollian Schwarzschild solution possesses a non-trivial temperature and entropy $\rightarrow$ Thermal state.

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- Carrollian Kerr?

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- Relation with the Carrollian gravitational theories of Hartong and Bergshoeff, Gomis, Rollier, Rosseel, ter Veldhuis?