# BMS symmetry in light-cone gravity: A null-front Hamiltonian study

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# BMS symmetry at a glance

**Bondi approach:**
BMS as the asymptotic symmetry group at null infinity

- infinite-dimensional enhancement of Poincaré group
- further extended to include superrotations, *Diff*$(\mathbb{S}^2)$, near-horizon symmetries
- connections to soft theorems and on-shell amplitudes, Celestial Holography etc.

[**Bondi-van der Burg-Metzner-Sachs '62**, Barnich-Troessaert, Hawking-Perry-Strominger Compère, Detournay, Grumiller, Sheikh-Jabbari, Campiglia-Laddha, Donnay, Zwikel and many more]

**Conformal Carroll approach:**

- BMS group as conformal Carroll group
- Further extensions to other Carrollian structures
- Symmetries of null hypersurfaces, Carrollian field theory

[**Duval-Gibbons-Hovarthy '14**, Ciambelli, Freidel, Flanagan, Leigh, Obers, Petropoulos, ...]

**Hamiltonian approach:**
BMS symmetry at spatial infinity

- Based on "$3+1$" Hamiltonian formulation of gravity à la ADM
- Canonical realization of the BMS algebra from an action principle
- a precursor to any quantization methods

[**Henneaux-Troessaert '18**, Fuentealba, Matulich, Tanzi, Guilini, ...]

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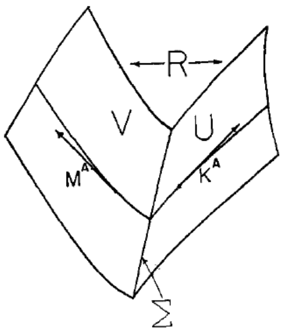


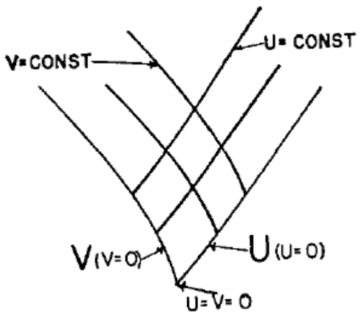


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- Unconstrained Hamiltonian systems
- Gravitational d.o.f. identified with the "conformal two-metric"

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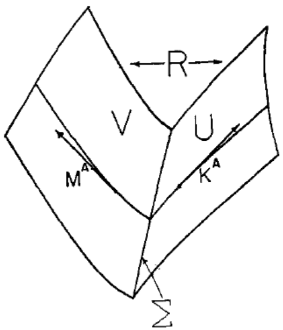


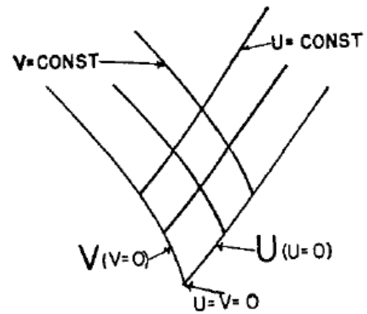


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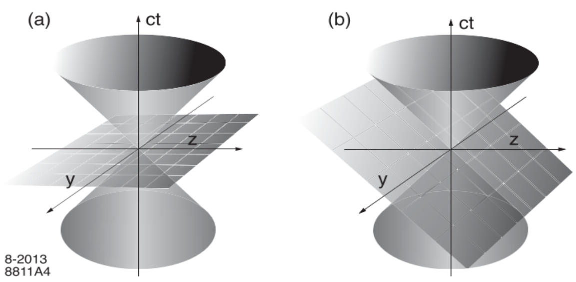
## Focus of this talk: BMS symmetry in the other Hamiltonian formulation

- Set up the 2+2 Hamiltonian formulation of light-cone gravity
- Study the BMS symmetry from residual gauge invariance

## Light-cone coordinates: The front-form

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"Forms of relativistic dynamics" [Dirac '49]

($a$) Instant form: time $x^0$

Initial data on a spatial hyperplane ($x^0 = 0$)

($b$) Front form: time $x^+ = \frac{x^0+x^3}{\sqrt{2}}$

Initial data on a null hyperplane ($x^+ = 0$)

- Poincaré generators in the instant form: $(P_\mu, M_{\mu\nu})$

$$[P, P] \sim 0 \ , \quad [P, M] \sim P \ , \quad [M, M] \sim M$$

$(P_0, M_{0i}) \ \rightarrow$ four "Hamiltonians"

- Poincaré generators in light-cone coordinates, $x^\mu = (x^+, x^-, x^i) \, , \quad i = 1, 2$

Kinematical $K = \{P_i, P_-, M_{ij}, M_{+-}\}$ ,

Dynamical $D = \{P_+, M_{i+}\} \rightarrow$ three "Hamiltonians" pick up corrections

Non-linear terms in $\mathbb{D}$ generators give us a handle on the dynamics of the interacting theory

# Light-cone Poincaré algebra in $d = 4$

- Non-vanishing commutators of the Poincaré algebra

$$J^+ = \frac{J^{+1} + iJ^{+2}}{\sqrt{2}} , \quad \bar{J}^+ = \frac{J^{+1} - iJ^{+2}}{\sqrt{2}} , \quad J = J^{12} , \quad H = P_+ = -P^- .$$

$$\begin{aligned} [H, J^{+-}] &= -iH , & [H, J^+] &= -iP , & [H, \bar{J}^+] &= -i\bar{P} \\ [P^+, J^{+-}] &= iP^+ , & [P^+, J^-] &= -iP , & [P^+, \bar{J}^-] &= -i\bar{P} \\ [P, \bar{J}^-] &= -iH , & [P, \bar{J}^+] &= -iP^+ , & [P, J] &= P \end{aligned}$$

... and many more

[Bengtsson-Bengtsson-Brink '83]

- Underlying Carrollian algebra

Rotation $\mathbb{J} = \{J^{12}, J^{+-}, J^+, \bar{J}^+\}$ , Boosts $\mathbb{K} = \{J^-, \bar{J}^-\}$

Translations $\mathbb{P} = \{P, \bar{P}, P_-\}$ , Hamiltonian $\mathbb{H} = P_+$

$$\begin{aligned} &[\mathbb{J}, \mathbb{J}] = \mathbb{J} , \quad [\mathbb{J}, \mathbb{P}] = \mathbb{P} , \quad [\mathbb{J}, \mathbb{K}] = \mathbb{K} \\ &[\mathbb{J}, \mathbb{H}] = 0 , \quad [\mathbb{H}, \mathbb{P}] = 0 , \quad [\mathbb{H}, \mathbb{K}] = 0 \\ &[\mathbb{P}, \mathbb{P}] = 0 , \quad [\mathbb{K}, \mathbb{K}] = 0 , \quad [\mathbb{P}, \mathbb{K}] = \mathbb{H} \end{aligned}$$

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- In terms of the Kinematical-Dynamical split

$$\mathbb{K} = \{P_i, P_-, M_{ij}, M_{+-}\} , \quad \mathbb{D} = \{P_+, M_{i+}\}$$

$$[\mathbb{K}, \mathbb{K}] = \mathbb{K} , \quad [\mathbb{K}, \mathbb{D}] = \mathbb{D} , \quad [\mathbb{D}, \mathbb{D}] = 0$$

# Gravity in the light-cone gauge

**"$lc_2$ formalism"** [Scherk-Schwarz, Schwarz-Goroff, Bengtsson-Cederwall-Lindgren]

- Einstein-Hilbert action in 4D

$$\mathcal{S} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\; R[g_{\mu\nu}]$$

- Light-cone gauge: Set the "minus" components to zero

$$g_{--} \;=\; g_{-i} \;=\; 0, \quad (i = 1, 2) \qquad \boxed{10 - 3 = 7}$$

  Parametrization

$$g_{+-} \;=\; -e^{\phi}, \qquad g_{ij} \;=\; e^{\psi}\gamma_{ij}$$

  $\phi$,$\psi$,$\gamma_{ij}$ are real and det $\gamma_{ij} = 1$

## Light-cone gravity metric

$$dS^2_{LC} \;=\; g_{\mu\nu}dx^{\mu}dx^{\nu} \;=\; -2e^{\phi}dx^{+}dx^{+} \;+\; g_{++}(dx^{+})^2 \;+\; g_{+i}dx^{+}dx^{i} \;+\; e^{\psi}\,\gamma_{ij}dx^{i}dx^{j}$$

given in terms of 7 functions $\{\phi, \psi, \gamma_{ij}, g_{++}, g_{+i}\}$

- "2+2" split of the Einstein field equations $R_{\mu\nu} = 0$ [Sachs, d'Inverno-Smallwood, ...]

  Dynamical equations: $R_{ij} = 0$

  Constraint equations: $R_{--} = R_{-i} = 0$

  Subsidiary equations: $R_{++} = R_{+i} = 0$

  Trivial equations: $R_{+-} = 0$

**Can we solve the constraint equations?**
$\rightarrow$ subject to choice of coordinates, gauge conditions and iterative integration schemes

- Constraint equation $R_{--} \; = \; 0$

$$2\,\partial_-\phi\,\partial_-\psi \;-\; (\partial_-\psi)^2 \;-\; 2\partial_-{}^2\psi \;+\; \frac{1}{2}\,\partial_-\gamma^{ij}\,\partial_-\gamma_{ij} \;=\; 0\,.$$

*Fourth* gauge choice : [Scherk-Schwarz]

$$\phi = \frac{\psi}{2} \qquad \boxed{7-1=6}$$

allows us to integrate † out $\psi$

$$\psi \;=\; \frac{1}{4}\frac{1}{\partial_-{}^2}(\partial_-\gamma^{ij}\,\partial_-\gamma^{ij}) \qquad \boxed{6-1=5}$$

† Our *integration scheme*

$$\partial_- f(x^-) = g(x^-) \quad \Rightarrow \quad f(x^-) = \frac{1}{\partial_-}g(x^-) = -\int \epsilon(x^- - y^-)\; g(y^-)\; dy^- + \text{“}\textit{constant}\text{”}$$

↑ FIXED BY INITIAL DATA

- The constraint $R_{-i} = 0$ eliminates $g_{+i}$ $\qquad \boxed{5-2=3}$

- $R_{-+} \; = \; 0$ allows us to eliminates $g_{++}$ $\qquad \boxed{3-1=2}$

# Light-cone action for gravity

- Closed form expression

$$S[\gamma_{ij}] = \frac{1}{2\kappa^2}\int d^4x\, e^{\psi}\left(2\,\partial_+\partial_-\phi + \partial_+\partial_-\psi - \frac{1}{2}\,\partial_+\gamma^{ij}\partial_-\gamma_{ij}\right)$$
$$-e^{\phi}\gamma^{ij}\left(\partial_i\partial_j\phi + \frac{1}{2}\partial_i\phi\partial_j\phi - \partial_i\phi\partial_j\psi - \frac{1}{4}\partial_i\gamma^{kl}\partial_j\gamma_{kl} + \frac{1}{2}\partial_i\gamma^{kl}\partial_k\gamma_{jl}\right)$$
$$-\frac{1}{2}e^{\phi-2\psi}\gamma^{ij}\frac{1}{\partial_-}R_i\frac{1}{\partial_-}R_j\ ,$$

where

$$R_i \equiv e^{\psi}\left(\frac{1}{2}\partial_-\gamma^{jk}\partial_i\gamma_{jk} - \partial_-\partial_i\phi - \partial_-\partial_i\psi + \partial_i\phi\partial_-\psi\right) + \partial_k(e^{\psi}\,\gamma^{jk}\partial_-\gamma_{ij})$$

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- Perturbative expansion

$$\gamma_{ij} \;=\; (e^{\kappa H})_{ij}\;, \quad H = \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix};\quad h_{22} \;=\; -h_{11}$$

Define

$$h \;=\; \frac{1}{\sqrt{2}}\,(h_{11} + i\; h_{12})\;, \quad \bar{h} \;=\; \frac{1}{\sqrt{2}}\,(h_{11} - i\; h_{12})$$

- Light-cone Lagrangian

$$\mathcal{L} = \frac{1}{2}\bar{h}\,\Box\,h \;+\; 2\kappa\;\bar{h}\;\partial_-{}^2\left(\frac{\bar{\partial}}{\partial_-}h\frac{\bar{\partial}}{\partial_-}h - h\frac{\bar{\partial}^2}{\partial_-{}^2}h\right) \;+ c.c. \;+\; \text{higher order terms}$$

$h$ and $\bar{h}$ represent gravitons of helicity +2 and -2 respectively

# Light-cone Hamiltonian for gravity

- Conjugate momenta (recall: $x^+$ is time)

$$\mathcal{L} = \bar{h}\,(\partial_-\partial_+ - \partial\,\bar{\partial})\,h \; + \; 2\kappa\;\bar{h}\;\partial_-{}^2\left(\frac{\bar{\partial}}{\partial_-}h\frac{\bar{\partial}}{\partial_-}h - h\frac{\bar{\partial}^2}{\partial_-{}^2}h\right)\; + \cdots$$

$$\pi \;=\; \frac{\delta\mathcal{L}}{\delta(\partial_+ h)} = -\,\partial_-\bar{h}\,, \qquad \bar{\pi} \;=\; \frac{\delta\mathcal{L}}{\delta(\partial_+\bar{h})} = -\,\partial_- h$$

$(\pi, \bar{\pi})$ not independent variables $\Rightarrow$ Half the d.o.f than in the 3+1 formalism

$\rightarrow$ a feature of *all* null-front Hamiltonian systems

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- Light-cone Hamiltonian for gravity

$$\mathcal{H} \;=\; \partial\bar{h}\,\bar{\partial}h \;+\; 2\,\kappa\,\partial_-^2\,\bar{h}\left(h\,\frac{\bar{\partial}^2}{\partial_-^2}h \;-\; \frac{\bar{\partial}}{\partial_-}h\,\frac{\bar{\partial}}{\partial_-}h\right) + c.c. \;+\; \mathcal{O}(\kappa^2)$$

- Poisson brackets

$$[h(x),\pi(y)] \;=\; \delta(x^- - y^-)\delta^{(2)}(x-y) \;\Rightarrow\; [h(x),\bar{h}(y)] \;=\; \underbrace{\frac{1}{\partial_-}\delta(x^- - y^-)}_{\text{STEP FUNCTION}}\,\delta^{(2)}(x-y)\,,$$

$$[\,h(x), h(y)\,] \;=\; [\,\bar{h}(x),\bar{h}(y)\,] \;=\; 0\,.$$

[Scherk-Schwarz' 75, Bengtsson-Cederwall-Lindgren '83]

# Symmetries of light-cone gravity

## Notion of symmetry

A canonical transformation $(h, \bar{h}) \xrightarrow{\delta_X} (\tilde{h}, \tilde{\bar{h}})$

which leaves the action invariant

$$\delta_X \mathcal{S}[h, \bar{h}] = 0$$

Transformation laws = P.B. with the generator $G_X[h, \bar{h}]$, e.g.

$$\delta_X h = \{ \, G_x, h \, \}_{PB}$$

For example,

Poincaré generators in terms of the fields $h$ and $\bar{h}$

$$H = P_+ = \int d^3x \, \mathcal{H}(h, \bar{h}) \, , \quad P = \int d^3x \partial_- \bar{h} \, \partial h \, , \quad P_- = d^3x \partial_- \bar{h} \partial_- h \, , \quad \cdots$$

$$J = i \int d^3x \partial_- \bar{h} \, (x\bar{\partial} - \bar{x}\partial - 2)h \, ,$$

$$J^- = \int d^3x [x\mathcal{H}(h, \bar{h}) + \partial_- \bar{h} \, (x^- \partial - 2\frac{\partial}{\partial_-})h + \mathcal{S}] \, , \quad \cdots$$

→ canonical realization of Poincarè algebra in light-cone gravity

[Bengtsson-Bengtsson-Brink]

# BMS symmetry from residual gauge invariance

Is there any residual reparameterization freedom left?

$$x^\mu \to x^\mu \; + \xi^\mu$$

First gauge condition holds

$$g_{--} = 0 \quad \Rightarrow \quad \partial_- \xi^+ = 0 \quad \Rightarrow \quad \xi^+ = f(x^+, x^j)$$

Second gauge condition $g_{-i} = 0$ leads to a relation between $\xi^+$ and $\xi^i$

$$\partial_- \xi^j \, g_{ij} \; + \; \partial_i \xi^+ \, g_{+-} = 0$$

Fourth gauge condition fixes $x^+$ dependence of $f(x^+, x^j)$

$$\begin{aligned}
\xi^+ &= f \; = \; \frac{1}{2} x^+ \partial_i Y^i + T(x^k) \\
\xi^i &= -\partial_k f \frac{1}{\partial_-}(g_{-+} g^{ik}) + Y^i(x^k) \\
\xi^- &= -\partial_i Y^i x^- + (\partial_+ \xi_i) x^i
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ONLY INDEPENDENT PARAMETER

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\end{aligned}$$

$\rightarrow$ leaves the light-cone gravity action invariant

$$\delta_\xi \mathcal{S}[h, \bar{h}] = 0$$

**iff** $\partial^2 Y = 0 \rightarrow$ only Lorentz rotations (**no supertranslations**)

[Ananth, Brink and SM]

# BMS algebra in light-cone gravity

- BMS transformation law (on the initial surface $x^+ = 0$),

$$\begin{aligned}\delta_{Y,\overline{Y},T}\, h \quad &= \quad Y(x)\,\bar{\partial}h + \overline{Y}(\bar{x})\,\partial h \; + (\partial\overline{Y} - \bar{\partial}Y)\,h + T\,\frac{\partial\bar{\partial}}{\partial_-}\,h \\ &\quad - 2\,\kappa\,T\,\partial_-\left(h\,\frac{\bar{\partial}^2}{\partial_-^2}h \;-\; \frac{\bar{\partial}}{\partial_-}h\,\frac{\bar{\partial}}{\partial_-}h\right) \;-\; 2\,\kappa\,T\,\frac{1}{\partial_-}\left(\frac{\partial^2}{\partial_-^2}\bar{h}\,\partial_-^2\,h\right) \\ &\quad - 2\,\kappa\,T\,\frac{\partial^2}{\partial_-^3}\,(\bar{h}\,\partial_-^2\,h) \;+\; 4\,\kappa\,T\,\frac{\partial}{\partial_-^2}\left(\frac{\partial}{\partial_-}\bar{h}\,\partial_-^2\,h\right) \;+ \mathcal{O}(\kappa^2)\end{aligned}$$

- Symmetry algebra

$$\Big[\,\delta(Y_1, \overline{Y}_1, T_1)\,,\; \delta(Y_2, \overline{Y}_2, T_2)\Big]\; h \quad = \quad \delta(Y_{12}, \overline{Y}_{12}, T_{12})\, h\ ,$$

with parameters

$$\begin{aligned} Y_{12} \quad &\equiv \quad Y_2\,\bar{\partial}\,Y_1 \;-\; Y_1\,\bar{\partial}\,Y_2 \\ \overline{Y}_{12} \quad &\equiv \quad \overline{Y}_2\,\partial\,\overline{Y}_1 \;-\; \overline{Y}_1\,\partial\,\overline{Y}_2 \\ T_{12} \quad &\equiv \quad [Y_2\,\bar{\partial}\,T_1 \;+\; \overline{Y_2}\,\partial\,T_1 \;+ \frac{1}{2}\,T_2(\bar{\partial}Y_1 \;+\; \partial\overline{Y}_1)] \;-\; (1 \leftrightarrow 2)\ . \end{aligned}$$

$\rightarrow$ BMS algebra in four dimensions

- Canonical generator for supertranslations

$$\delta_T h \;=\; [G_T\,, h]\,, \qquad\qquad \delta_T\bar{h} \;=\; [G_T\,, \bar{h}]\,.$$

$$G_T \quad = \quad \int d^3x\,\partial_-\bar{h}\,(\delta_T h) \;=\; \int d^3x\,\partial_-\bar{h}\;T\,\frac{\partial\bar{\partial}}{\partial_-}h \;+\; \mathcal{O}(\kappa)\,,$$

# Light-cone representation of the BMS algebra

- Light-cone Poincaré algebra

$$\mathbb{K}: \quad \{P, \bar{P}, P^{+}, J, J^{+}, \bar{J}^{+}, J^{+-}\}$$
$$\mathbb{D}: \quad \{P^{-} \equiv H, J^{-}, \bar{J}^{-}\}$$

$$[\,\mathbb{K}, \mathbb{K}\,] \;=\; \mathbb{K}\,, \quad [\,\mathbb{K}, \mathbb{D}\,] \;=\; \mathbb{D}\,, \quad [\,\mathbb{D}, \mathbb{D}\,] \;=\; 0\,.$$

- Light-cone BMS algebra

$$\mathbb{K} \;\rightarrow\; \mathbb{K}\,,$$
$$\mathbb{D} \;\rightarrow\; \mathbb{D}(T)\,,$$

$$[\,\mathbb{K}, \mathbb{K}\,] \;=\; \mathbb{K}\,, \quad [\,\mathbb{K}, \mathbb{D}(T)\,] \;=\; \mathbb{D}(T)\,, \quad [\,\mathbb{D}(T), \mathbb{D}(T)\,] \;=\; 0\,.$$

Dynamical part enhanced to infinite-dimensional supertranslations
labeled by a single parameter T

- Poincaré part of the BMS

$$\partial^2 T = \bar{\partial}^2 T = 0$$

$\Rightarrow \mathbb{D}(T)$ reduces to $\mathbb{D}: \{H, J^{-}, \bar{J}^{-}\} \quad \rightarrow$ the three "Hamiltonians" of Dirac

$a\partial_+ \quad i b x \partial_+ \quad -i\bar{b}\bar{x}\partial_+$

[Ananth, Brink and SM]

# Outlook

Light-cone BMS algebra: key features

- BMS from residual gauge invariance in the bulk: No reference to asymptotic limits
- Dynamical part of the algebra enhanced to infinite-dimensional supertranslations
- Lorentz invariance of the Hamiltonian eliminates superrotations

Some open questions

- How is the light-cone initial data is related to null infinity data?
- Connections to on-shell amplitudes and geometry on null hypersurfaces
- Self-dual and anti self-dual gravity; double copy results

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"TRUE" MORE THAN NOT ONLY AT

BMS symmetry as the asymptotic symmetry group at null infinity

"TRUE" symmetry:
- INV. OF ACTION
- CANONICAL P.B.

MORE THAN asymptotic:
- CONF. CARROLL
- RESIDUAL GAUGE INV.

NOT ONLY AT null infinity:
- SPATIAL INFINITY
- NULL HYPERSURFACES
- NEAR BH HORIZON