Carrollian Archeology

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Université de Nice

…notwithstanding the sagacious advice by Lewis Carroll, who wrote :
“It’s no use going back to yesterday, because I was a different person then.”

Some other Carroll groups in science…

Some other Carroll groups in business…

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CARROLL
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Possible sponsors for next Carrollian conferences?

I. Paying tributes…

Paul Langevin (1872-1945) 150th birthday !

« L'évolution de l'espace et du temps »,
Congrès international de philosophie, Bologne, 1911 ;
Scientia X (19), 31-54 (1911)
https://en.wikisource.org/wiki/Translation:The_Evolut
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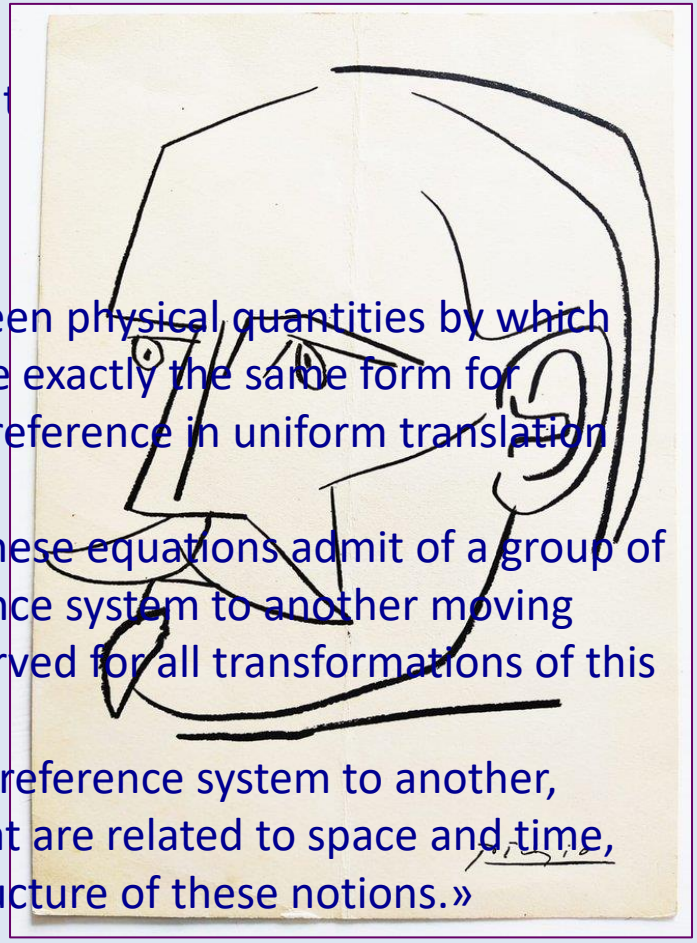


«It is's an experimental fact that the equations between physical quantities by which we translate the laws of the outside world, must have exactly the same form for different groups of observers, for various systems of reference in uniform translation relative to each other.
This requires, in the language of mathematics, that these equations admit of a group of transformations corresponding to a change of reference system to another moving relative to it. The equations of physics must be preserved for all transformations of this group.
In such a transformation, when one moves from one reference system to another, measures of various magnitudes, especially those that are related to space and time, are changed in a manner that corresponds to the structure of these notions.»

Paul Langevin (1872-1945)

Einstein's obituary to Langevin (1947):
«It appears to me as a foregone conclusion that he would have developed the special relativity theory, had not that been done elsewhere.»

Eugene P. Wigner (1902-1995)

— *Gruppentheorie und ihre Anwendungen auf die Quantenmechanik der Atomspektren*, Vieweg Verlag, 1931.

Group Theory and its Application to the Quantum Mechanics of Atomic Spectra, Academic Press, 1959.

— « On unitary representations of the inhomogeneous Lorentz group », *Annals of Mathematics* 40, 1939, p. 141.

— [with E. Inönu] « On the Contraction of Groups and Their Representations », *Proc. Natl. Acad. Sci.* 39 (6), 1953, 510–24.

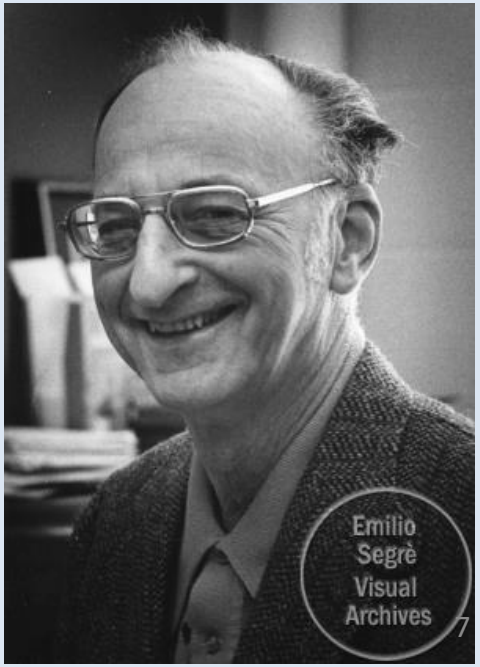
Eugene P. Wigner (1902–1995)

Lev Pontrjagin (1902-1995)

Topological Groups, University of California Press, 1939

Morton Hamermesh (1915-2003)

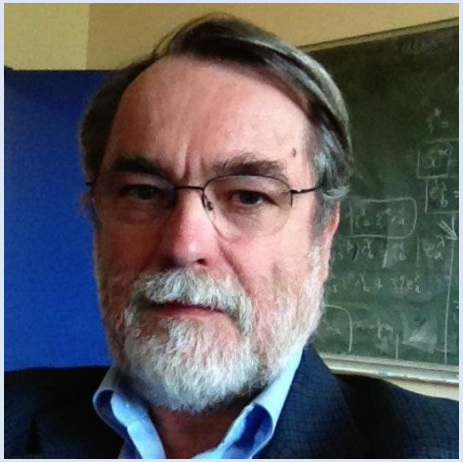
Group Theory and Its Applications to Physical Problems, Addison-Wesley, 1952



Louis Michel
(1923-1999)

François Lurçat
(1927-2012)

Henri Bacry
(1928-2010)



Christian Duval
(1947-2018)

II. The birth of the Carroll group

The birth of the Carroll group

Ann. Inst. Henri Poincaré,
Vol. III, n° 1, 1965, p. 1-12.

Section A :
Physique théorique.

Une nouvelle limite non-relativiste du groupe de Poincaré

par

Jean-Marc LÉVY-LEBLOND
(Laboratoire de Physique Théorique, Orsay).

Begetting the Carroll group

Teaching relativity…

1965
C = 1

$$\text{Lorentz} \quad \begin{cases} \Delta x' = \gamma(\Delta x - v\Delta t) \\ \Delta t' = \gamma(\Delta t - v\Delta x) \end{cases} \quad \text{where} \quad \gamma = \sqrt{1 - v^2}$$

$$\text{if} \quad v << 1$$

if $\Delta t >> \Delta x$

(large time-like)

$$\text{Galilei} \quad \begin{cases} \Delta x' = \Delta x - v\Delta t \\ \Delta t' = \Delta t \end{cases}$$

what if $\Delta t << \Delta x$?

(large space-like)

$$\text{Carroll} \quad \begin{cases} \Delta x' = \Delta x \\ \Delta t' = \Delta t - v\Delta x \end{cases}$$

better
$c \neq 1$

Lorentz
$$\begin{cases} \Delta x' = \gamma(\Delta x - \frac{v}{c} c\Delta t) \\ \Delta t' = \gamma(\Delta t - \frac{v}{c} c^{-1}\Delta x) \end{cases} \quad \text{where} \quad \gamma = \sqrt{1 - v^2 / c^2}$$

if $v / c << 1$

if $\Delta t >> c^{-1}\Delta x$ $("c \to \infty")$

Galilei
$$\begin{cases} \Delta x' = \Delta x - v\Delta t \\ \Delta t' = \Delta t \end{cases}$$

but if $\Delta t << c^{-1}\Delta x$? $("c \to 0")$

Carroll
$$\begin{cases} \Delta x' = \Delta x \\ \Delta t' = \Delta t - v\Delta x \end{cases}$$

JMLL, *Ann. Inst. Henri Poincaré*, 1965

Abstract.

« … the transformation properties of great space-like intervals are described by a group which is a new non-relativistic limit of the Poincaré group.

The group is distinct from the Galilei group but is also obtained from the Poincaré group by a contraction.

… In a world whose invariance group would be this new group, there would be practically no causality. We thus suggest the name “Carroll group” for this degenerate cousin of the Poincaré group.

The purpose of this paper is mainly pedagogical… »

JMLL, *Ann. Inst. Henri Poincaré*, 1965

Conclusion

«The effective usefulness of the Carroll group seems doubtful for the moment.

…

Nevertheless, theoretical physics has recently shown itself to be friendly enough for many groups with a limited physical interest; this is why I have not too much scruple in bringing to light this degenerate cousin of the Poincaré group. »

…and Sen Gupta

IL NUOVO CIMENTO VOL. XLIV A, N. 2 21 Luglio 1966

On an Analogue of the Galilei Group.

N. D. SEN GUPTA

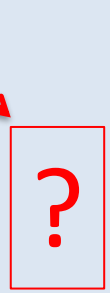
Tata Institute of Fundamental Research - Bombay

(ricevuto il 6 Febbraio 1965; manoscritto revisionato ricevuto il 24 Febbraio 1966)

Sen Gupta

«Introduction

As a complement to the fact that the transformation of the special theory of relativity in the limit of small velocity compared to that of light reduces to the Galilei transformation, one may enquire from a purely formal standpoint whether a similar limit exists for (…) a velocity large in comparison to the velocity of light.»



Sen Gupta

«3. The group U and the Lorentz group

In this Section, we propose to show that a Lorentz transformation contains a suitable pair of transformations, one each from U and G.»

$$\begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -v & 1 \end{pmatrix} \times \begin{pmatrix} \gamma & 0 \\ 0 & \gamma^{-1} \end{pmatrix} \times \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix}$$

Lorentz transform. | Carroll transform. | scale transform. | Galilei transform.

adapted and corrected from Sen Gupta

Sen Gupta

«Discussion

The decomposition of Lorentz transformations (…) points to the complementary aspects of the groups $\mathbb{G}$ and $\mathbb{U}$ with respect to space and time.

Although the group $\mathbb{U}$ has no direct significance (…), yet the appearance of an element of $\mathbb{U}$ in association with an element of $\mathbb{G}$ in the Lorentz transformation may be of some physical interest.»

[work in progress, JMLL]

III. A terminological excursus

A terminological excursus
« Non-relativistic » ?

Dictionaries (Merriam-Webster, Collins) :

« Nonrelativistic : not based on or involving the theory of relativity. »

First known use in 1930.

But it has finally become clear that the **principle** of relativity allows for several possible **theories**, especially the modern "Einsteinian" one and the much older — although implicit for a long time — classical "Galilean" one, not to speak of the "Carrollian" one.

A terminological excursus
"non-relativistic" ?

Furthermore:
« [The theory of space-time] is an *Invariantentheorie* of the Lorentz group. **The name 'relativity theory' is an unfortunate choice**: the relativity of space and time is not the essential thing, which is the independence of the laws of nature from the point of view of the observer. » Sommerfeld, 1948

This independence/invariance is characterized by the intrinsic structure of spacetime, that is,what I believe natural to name a **chronogeometry.**

A proposal :
— replace "relativity" by "chronogeometry"
— replace "non-relativistic" by "Galilean" (…or "Carrollian")

A terminological excursus
“speed of light” ?

Einstein’s derivation of Lorentz tranformations (1905) was based on the so-called second postulate, that of the invariance of light velocity.

But Einsteinian chronogeometry is not intrinsically linked to the properties of light: indeed it rules as well non-electromagnetic phenomena, such as strong interactions.

And the Lorentz group may be constructed without any appeal to the second postulate (Ignatowsky 1911, Frank & Rothe1911), as many authors have rediscovered since).

Furthermore,suppose the photon has a non-zero mass, however small; then light would not travel with a non-invariant velocity…

A proposal :

— replace ”speed of light“ by “speed limit” or better: “Einstein constant”.

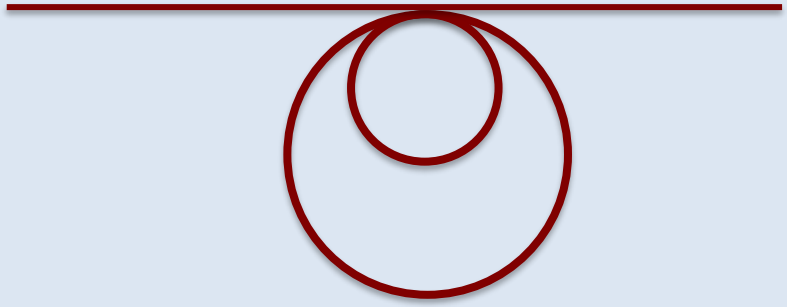
A terminological excursus
"group contraction" ?

E. Inonu & E. P. Wigner, « On the contraction of groups and their representations »,
Proc. Natl Acad. Sci. 39, 1953, pp. 510-524

«We shall call the operation of obtaining a new group by a singular transformation of the infinitesimal elements of the old group a contraction of the latter. The reason for this term will become clear below.
(…)
In the limit e = 0 (if such a limit exists), one will have contracted the whole group to an infinitesimally small neighborhood of the group. This justifies the name given to the process considered.»

A terminological excursus
“group contraction” ?

However, it is not really a proper formulation to say that « one will have contracted the whole group to an infinitesimally small neighborhood of it ».
Rather, a better statement would be that one will have stretched an infinitesimally small neigborhood of the group to a fully fledged new group.



A proposal:
— replace “goup contraction” by “group distension”

A terminological excursus

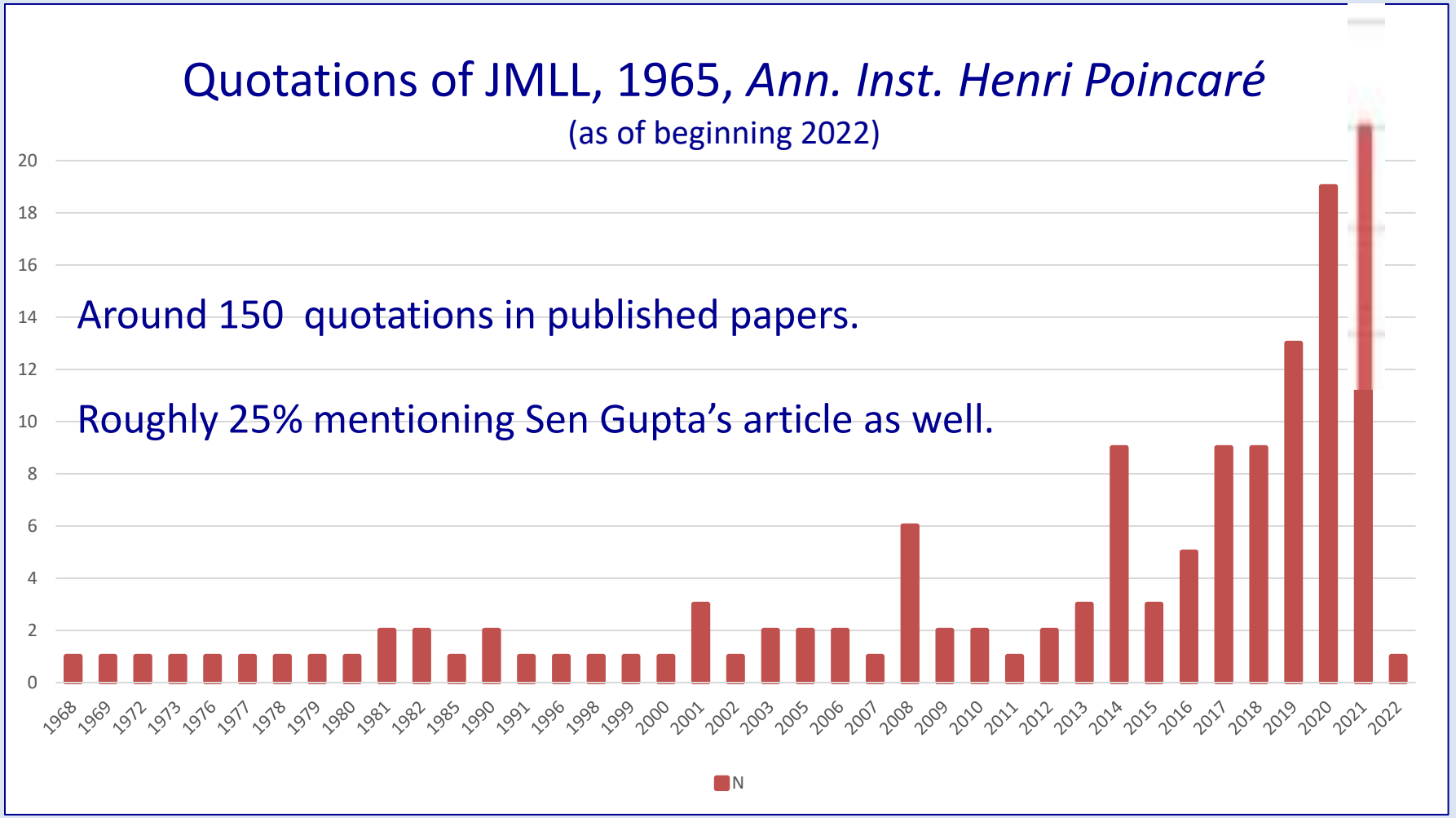
Why should we care about words since we have the formulas to rely on?

«Words do not convey only the sheer idea of the object signified, but also a more or less important number of related meanings and pictures.» Leopardi 1820

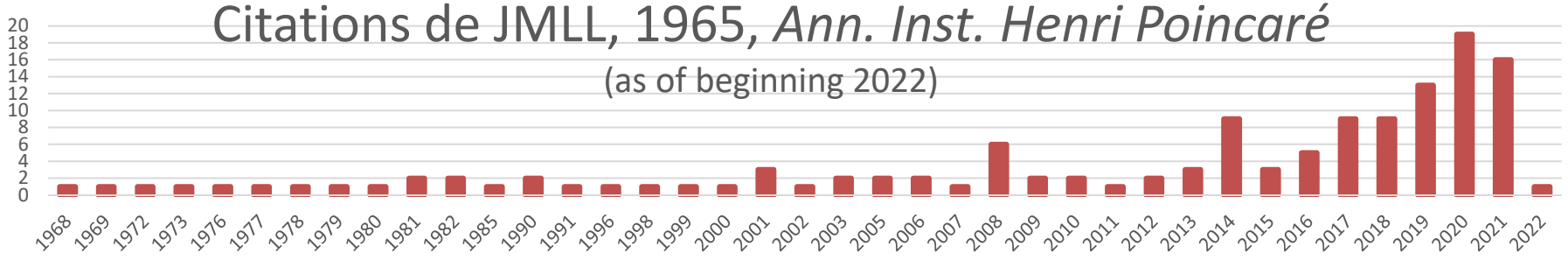
Research
(beyond formalism)
Teaching
(beyond technicalities)
Popularisation
(beyond catchwords)

IV. The late blooming of Carrollian physics

The late blooming of Carrollian physics



The late blooming of Carrollian physics



1965-2000 : 1 or 2 quotations/year in 20 or so journals, as varied as *J. Math.Phys., Ann. Der Phys., J. of Phys. Comm., Nuovo Cimento, Int. J. Theor. Phys., Bull. Acad. R. Belg., Phys. Lett., J. Geometry & Phys.*, etc.,
mainly about abstract Group theory, Special relativity, Electromagnetism.

Since 2000 (whith acceleration after 2014): concentration in *J. Math. Phys., J. of High Energy Phys., Class. & Qu. Gravity, Phys. Rev. D, Phys. Rev. Lett., General Relat. & Gravit., J. of Cosmology & Astroparticle, Phys. Lett.*,
mainly concerned with General relativity, Field Theory, Gravitation.

Possible Kinematics

Henri Bacry & Jean-Marc Lévy-Leblond
"Possible Kinematics", *J. Math. Phys.* 9, 1968, 1605-1614

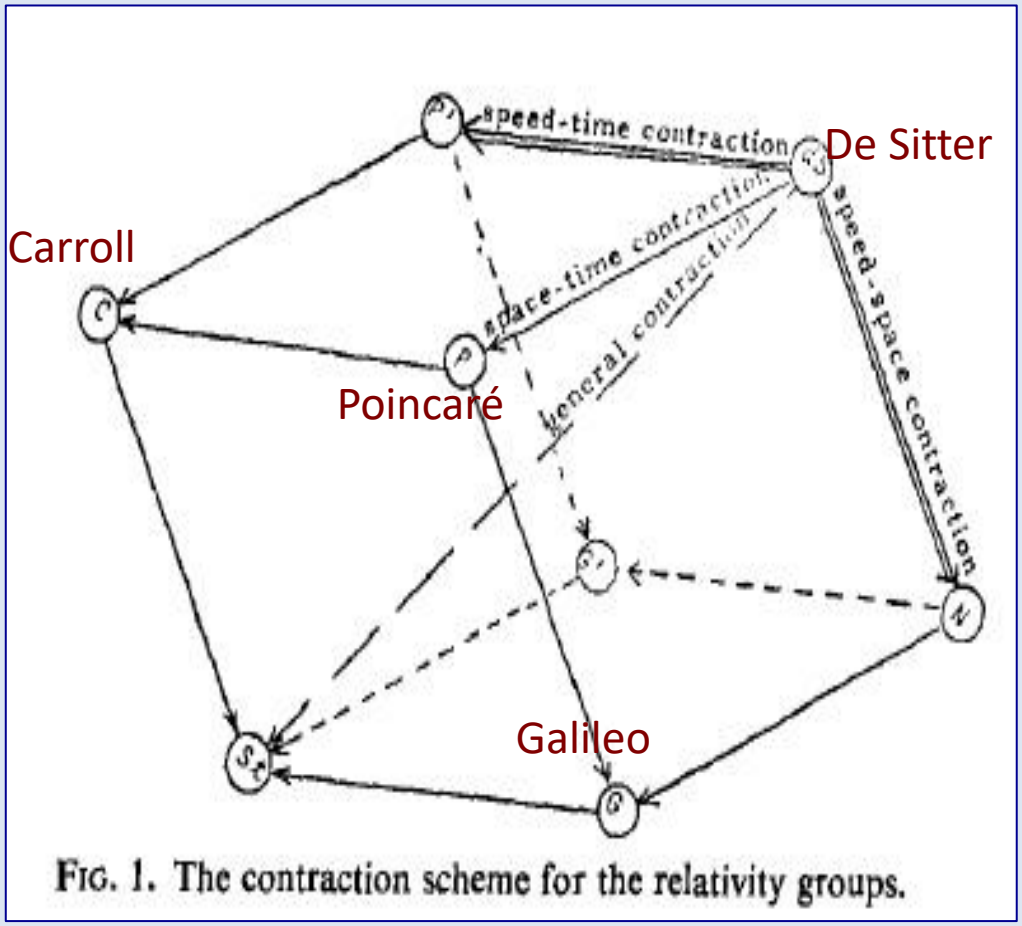
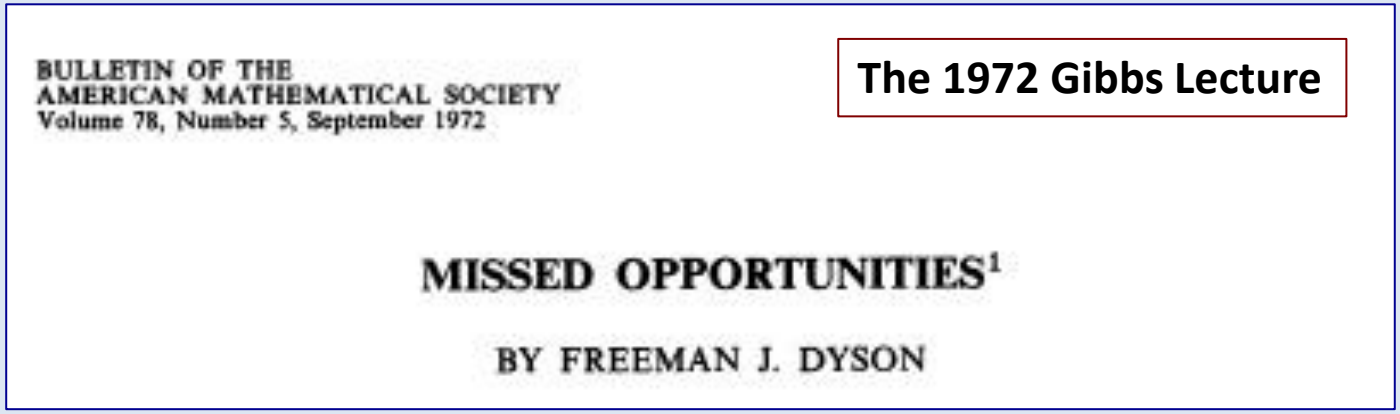


FIG. 1. The contraction scheme for the relativity groups.

Possible Kinematics



BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

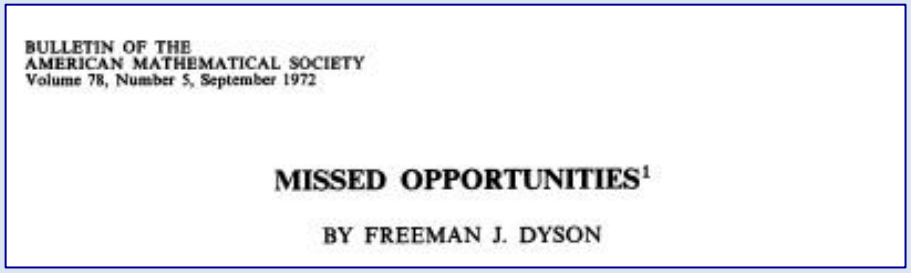
The 1972 Gibbs Lecture

MISSED OPPORTUNITIES[1]

BY FREEMAN J. DYSON

1. Introduction
2. Digression into number theory
3. The Maxwell equations
4. Kinematical groups.
5. Quaternions and vectors
6. General coordinate invariance
7. Feynman sums
8. Conclusion

Possible Kinematics



BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

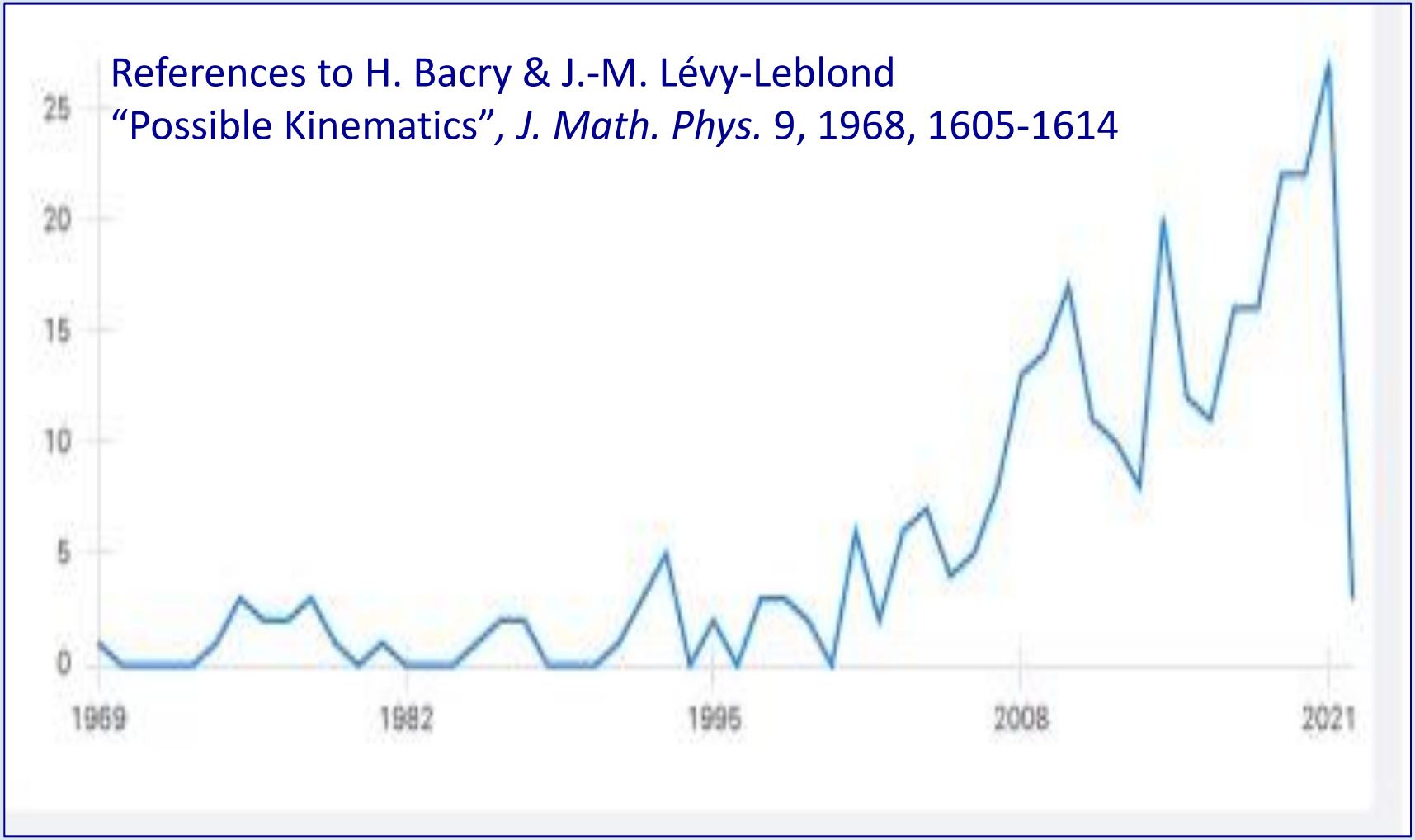
MISSED OPPORTUNITIES[1]

BY FREEMAN J. DYSON

« The eight groups can then be visualized as the vertices of a cube. P and G are the only kinematical groups that correspond to orthodox physical universes. But the other five groups are just as good, mathematically speaking.
The most interesting of the heterodox groups are N and C. N describes a Newtonian universe with curved space-time. C describes a universe in which space is absolute, in contrast to the Galilei group G which has time absolute.
The group C was discovered by Lévy-Leblond and called by him the Carroll group. In the Carroll universe, all objects have zero velocity although they may have nonzero momentum.
Carroll was a pure mathematician who had already foreseen this possibility in 1871: "A slow sort of country," said the Queen, "Now, here, you see, it takes all the running you can do, to keep in the same place. ” But his mathematical colleagues once again missed an opportunity by failing to take him seriously. »

Possible Kinematics

References to H. Bacry & J.-M. Lévy-Leblond
"Possible Kinematics", *J. Math. Phys.* 9, 1968, 1605-1614



V. A few conclusions

1. The pace of contemporary science is not always “fast and furious”.

2. Sharing knowledge may help developing it.

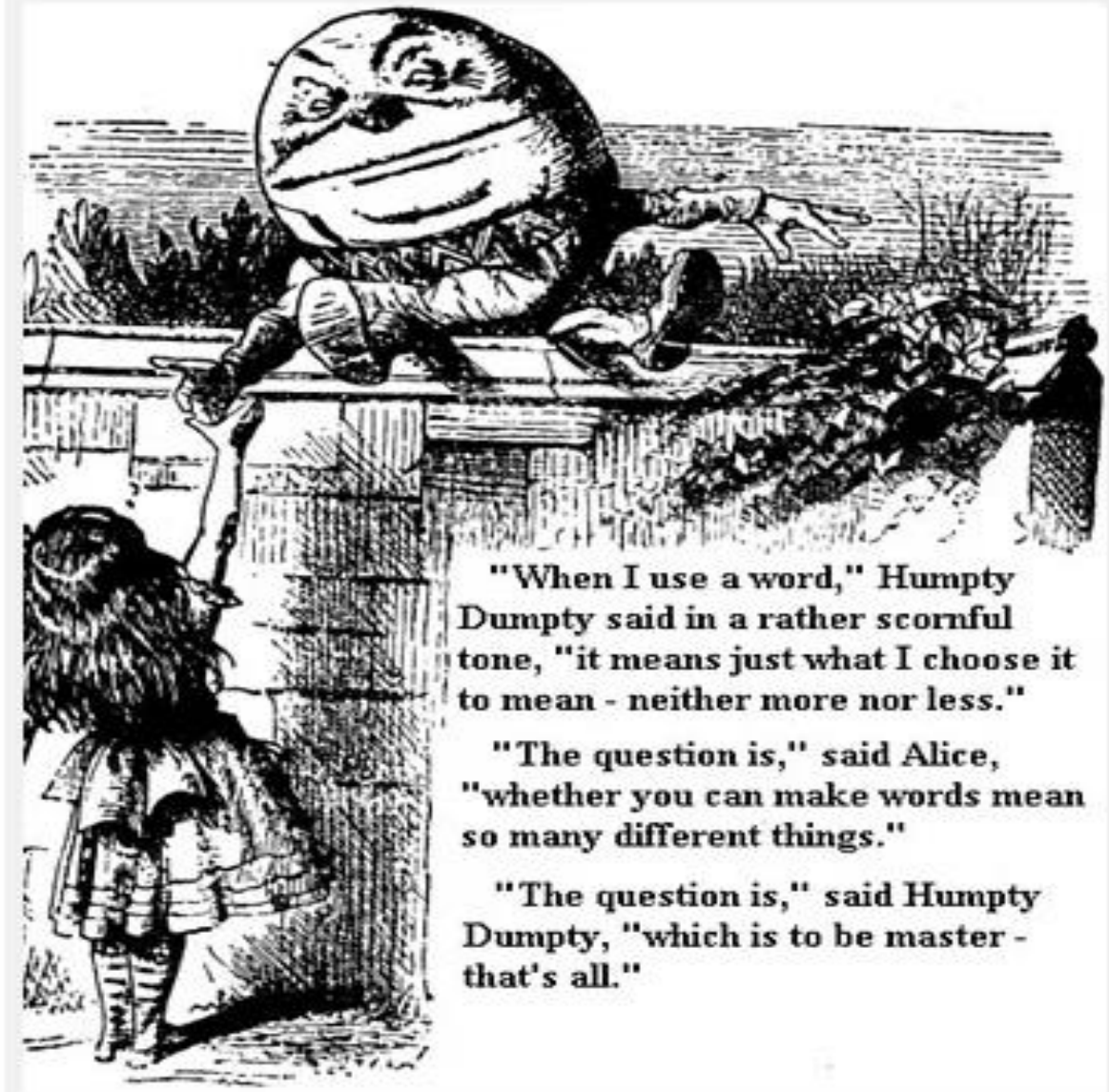
et l'enrichir

« L'homme ne peut jouir de son savoir qu'autant qu'il peut le communiquer à quelqu'un. »

Giacomo Casanova, *L'Icosaméron* (1788)

« One cannot enjoy one's knowledge and enrich it but by sharing it »

3. Language should be taken seriously



4. Missed opportunities

«Undoubtedly, there exist many more missed opportunties to create new branches of pure mathematics out of old problems of applied science.» (Dyson)

But no less undoubtedly, reciprocally and more generally, there exist many missed opportunities to solve new problems of science out of old branches of it.

Thanks for your attention, with my apologies, since “it's a poor sort of memory that only works backwards.” (Lewis Carroll — who else?)