Carroll-invariant field theories

Marc Henneaux

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while the other is the Carroll algebra ($c \to 0$, $B_i \to cB_i$, $E \to cE$),

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The purpose of this talk is to describe algebraic and geometrical properties of Carroll-invariant theories.

Causality in Galilean-invariant and Carroll-invariant theories

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More generally, dynamical Carroll-invariant field equations reduce to ordinary differential equations with respect to time.

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Generalization to p-forms, gravity, higher spins in arbitrary dimension : M. Henneaux and P. Salgado-Rebolledo, [arXiv :2109.06708 [hep-th]] (see also J. de Boer, J. Hartong, N. A. Obers, W. Sybesma and S. Vandoren [arXiv :2110.02319 [hep-th]]

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Note that both limits are compatible with gauge invariance.

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Null vectors can then be “normalized” through

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No extra structure (parallel transport, unique foliation by transverse hyperplanes etc) is introduced.

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For instance, the scalar product $G^{\alpha\beta}v_\alpha w_\alpha$ does not depend on the choice of θ_α if the covectors v_α and w_α are both transverse ($n^\alpha v_\alpha = 0 = n^\alpha w_\alpha$).

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$$(g_{\alpha\beta}) = \begin{pmatrix} 0 & 0 \\ 0 & I_{d\times d} \end{pmatrix}, \qquad (n^\alpha) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \Omega = 1$$

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$$G'^{\alpha\beta} \equiv \frac{\partial x'^\alpha}{\partial x^\mu}\frac{\partial x'^\beta}{\partial x^\nu} G^{\lambda\mu} = G^{\alpha\beta}(g_{\rho\sigma}, n^\tau, \theta'_\gamma \equiv \theta_\delta \frac{\partial x^\delta}{\partial x^\gamma}).$$

Sufficient conditions for a theory to be Carroll invariant

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$$E = \int d^d x \mathscr{E}(x), \qquad P_k = \int d^d x \mathscr{P}_k(x),$$

while the generators of Carroll boosts and spatial rotations read

$$B^k = \int d^d x x^k \mathscr{E}(x), \qquad M^{rs} = \int d^d x (x^r \delta^{sk} - x^s \delta^{rk}) \mathscr{P}_k(x).$$

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(In the Poincaré case, $[\mathscr{E}(x), \mathscr{E}(y)] \sim (\mathscr{P}^k(x) + \mathscr{P}^k(y))\delta_{,k}(x-y)$: Schwinger - Dirac)

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We consider for definiteness electromagnetism

$$S[A_i, \pi^i, A_t] = \int dt \left[\int d^d x \pi^a \dot{A}_a - H + \int d^d x A_t \partial_a \pi^a \right], \; H = \int d^d x \mathscr{E}^C$$

and start with the electric case ($\mathscr{E}^C = \mathscr{E}^E = \frac{1}{2}\pi^a \pi_a$).

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The integrand $G^{\rho\sigma} n^\alpha F_{\alpha\rho} n^\beta F_{\beta\sigma}$ is well-defined because $F_{\alpha\beta} n^\beta$ is transverse, $n^\alpha F_{\alpha\beta} n^\beta = 0$.

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$$S^M[A_\alpha,\pi^\beta,\theta_\gamma]=\int d^Dx\left(\pi^\alpha F_{\alpha\beta}n^\beta-\frac{1}{4}G^{\alpha\beta}G^{\rho\sigma}F_{\alpha\rho}F_{\beta\sigma}\right)$$

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The construction can be generalized to arbitrary p-forms.

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$$\mathscr{H} = G_{ijkm}\pi^{ij}\pi^{mn} - c^6 R\sqrt{g}, \qquad \mathscr{H}_i = -2\pi_i{}^j{}_{|j}.$$

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One can consistently take the limit $c \to 0$ since the constraints remain first class in the limit,

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&[\mathcal{H}_m(x), \mathcal{H}_k(x')] = \mathcal{H}_m(x')\delta_{,k}(x-x') + \mathcal{H}_k(x)\delta_{,m}(x-x')
\end{aligned}
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where

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and one sees therefore that $c \to 0$ (Carroll limit) is equivalent to $G \to \infty$ (strong coupling limit, Isham 1975) or $\epsilon = 0$ (zero Hamiltonian signature limit -Teitelboim 1978), keeping N^{resc} finite.

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(Action well-defined, more in Henneaux 1979)

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One now finds $\dot{g}_{ij} \sim 0$.

Ideals of the Carroll algebra

$$[M_i, M_j] = \epsilon_{ijk} M_k, \quad [M_i, B_j] = \epsilon_{ijk} B_k, \quad [B_i, B_j] = 0,$$
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(A. Pérez 2021 ; O. Fuentealba, M. Henneaux, Patricio Salgado-Rebolledo and J. Salzer, in preparation)

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THANK YOU !