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## Carroll-invariant field theories

### Marc Henneaux

Vienna - Carroll Workshop - 15 February 2022

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### The Poincaré algebra

$$\begin{split} [M_i, M_j] &= \epsilon_{ijk} M_k, \quad [M_i, B_j] = \epsilon_{ijk} B_k, \quad [B_i, B_j] = -\epsilon_{ijk} M_k, \\ [M_i, P_j] &= \epsilon_{ijk} P_k, \quad [P_i, B_j] = \delta_{ij} E, \quad [M_i, E] = 0 \quad [E, B_i] = P_i, \end{split}$$

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### The Poincaré algebra

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has (at least) two interesting contractions.

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### has (at least) two interesting contractions. One is the familiar Galilean algebra $(c \to \infty, B_i \to \frac{1}{c}B_i, E \to cE)$

$$[M_i, M_j] = \epsilon_{ijk} M_k, \quad [M_i, B_j] = \epsilon_{ijk} B_k, \quad [B_i, B_j] = 0, [M_i, P_j] = \epsilon_{ijk} P_k, \quad [P_i, B_j] = 0, \quad [M_i, E] = 0 \quad [E, B_i] = P_i,$$

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### has (at least) two interesting contractions. One is the familiar Galilean algebra ( $c \rightarrow \infty$ , $B_i \rightarrow \frac{1}{c}B_i$ , $E \rightarrow cE$ )

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while the other is the Carroll algebra ( $c \rightarrow 0, B_i \rightarrow cB_i, E \rightarrow cE$ ),

$$\begin{split} & [M_i,M_j] = \epsilon_{ijk}M_k, \quad [M_i,B_j] = \epsilon_{ijk}B_k, \quad [B_i,B_j] = 0, \\ & [M_i,P_j] = \epsilon_{ijk}P_k, \quad [P_i,B_j] = \delta_{ij}E, \quad [M_i,E] = 0 \quad [E,B_i] = 0. \end{split}$$

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It also appears in many other gravity-related contexts (BMS, cosmology, etc... this workshop !).

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It also appears in many other gravity-related contexts (BMS, cosmology, etc... this workshop !).

The purpose of this talk is to describe algebraic and geometrical properties of Carroll-invariant theories.

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Conclusions and comments (Based on Henneaux 1979 and Henneaux + Salgado-Rebolledo 2021)

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Conclusions and comments (Based on Henneaux 1979 and Henneaux + Salgado-Rebolledo 2021) The light cones determine the causality structure in Minkowski space, ruling Poincaré invariant field theories.

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Conclusions and comments (Based on Henneaux 1979 and Henneaux + Salgado-Rebolledo 2021)

The light cones determine the causality structure in Minkowski space, ruling Poincaré invariant field theories.

In the nonrelativistic limit, the light cones completely open to the hyperplanes  $x^0 = const$ .

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In the nonrelativistic limit, the light cones completely open to the hyperplanes  $x^0 = const$ .

It is the opposite in the Carrollian limit : the light cones collapse to the lines  $x^k = const$  generated by  $\frac{\partial}{\partial t}$ .

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The field at time *t* depends only on the field and a finite number of its time derivatives at time t = 0 at the same spatial point.

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("ultrarelativistic = ultralocal")

More generally, dynamical Carroll-invariant field equations reduce to ordinary differential equations with respect to time.

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# Just as in the case of the Galilean limit, there are two different types of Carroll contractions,

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### Just as in the case of the Galilean limit, there are two different types of Carroll contractions, the "electric" and the "magnetic" ones.

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$$S[A_{a_1\cdots a_p}, \pi^{a_1\cdots a_p}, A_{0a_1\cdots a_{p-1}}]$$
  
=  $\int d^D x \Big( \pi^{a_1\cdots a_p} \dot{A}_{a_1\cdots a_p} - A_{0a_2\cdots a_p} \mathcal{G}^{a_2\cdots a_p} - \mathcal{H} \Big),$ 

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where

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#### where

$$\mathcal{H} = \mathcal{E}^E + \mathcal{E}^M,$$

$$\mathcal{E}^E = \frac{p!c^2}{2} \pi_{a_1 \cdots a_p} \pi^{a_1 \cdots a_p}, \quad \mathcal{E}^M = \frac{1}{2(p+1)!} F_{a_1 \cdots a_{p+1}} F^{a_1 \cdots a_{p+1}}$$

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and

$$\mathscr{G}^{a_1\cdots a_{p-1}} = -p\partial_a \pi^{aa_1\cdots a_{p-1}}.$$

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### The magnetic limit is obtained by taking the direct limit $c \rightarrow 0$ and yields

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Conclusions and comments The magnetic limit is obtained by taking the direct limit  $c \rightarrow 0$ and yields

$$S = \int d^{D}x \Big( \pi^{a_1 \cdots a_p} \dot{A}_{a_1 \cdots a_p} - A_{0a_2 \cdots a_p} \mathcal{G}^{a_2 \cdots a_p} - \mathcal{E}^M \Big),$$

while the electric limit is obtained by first rescaling the fields as

$$A_{\mu_1\cdots\mu_p} \to cA_{\mu_1\cdots\mu_p}, \qquad \pi^{a_1\cdots a_p} \to \frac{1}{c} \pi^{a_1\cdots a_p}$$

and then taking the limit  $c \rightarrow 0$ , leading to

$$S = \int d^{D}x \Big( \pi^{a_1 \cdots a_p} \dot{A}_{a_1 \cdots a_p} - A_{0a_2 \cdots a_p} \mathcal{G}^{a_2 \cdots a_p} - \mathcal{E}^E \Big).$$

Note that both limits are compatible with gauge invariance.

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### Ingredients :

(i) Degenerate metric  $g_{\alpha\beta}$  of rank D-1 which is positive semi-definite, i.e. det  $g_{\alpha\beta} = 0$ ,  $g_{\alpha\beta}v^{\alpha}v^{\beta} \ge 0$ , with  $g_{\alpha\beta}v^{\alpha}v^{\beta} = 0$  if and only if the vector  $v^{\alpha}$  is along the null direction ("null vector")

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$$\mathscr{G}^{\alpha\beta} = \Omega^2 n^\alpha n^\beta$$

where  $\mathscr{G}^{\alpha\beta}$  are the minors of  $g_{\alpha\beta}$  ( $\mathscr{G}^{\alpha\beta}g_{\gamma\beta} = 0$  since det  $g_{\alpha\beta} = 0$ ).

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$$G^{\alpha\beta}g_{\beta\gamma}=\delta^{\alpha}_{\gamma}-n^{\alpha}\theta_{\gamma}.$$

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If one imposes in addition the condition  $G^{\alpha\beta}\theta_{\alpha}\theta_{\gamma} = 0$ , the tensor  $G^{\alpha\beta}$  is completely determined.

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Since the one-form  $\theta_{\alpha}$  comes on top of the basic Carroll structure defined by the degenerate metric  $g_{\alpha\beta}$  and density  $\Omega$ , we shall insist that "Carrollian physics" should *not* depend on  $\theta_{\alpha}$ . For instance, the scalar product  $G^{\alpha\beta}v_{\alpha}w_{\alpha}$  does not depend on the choice of  $\theta_{\alpha}$  if the covectors  $v_{\alpha}$  and  $w_{\alpha}$  are both transverse  $(n^{\alpha}v_{\alpha} = 0 = n^{\alpha}w_{\alpha})$ .

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### A flat Carroll structure is a vector space equipped with a Carroll inner product and a choice of normalization of the null vectors.

One can take

$$(g_{\alpha\beta}) = \begin{pmatrix} 0 & 0\\ 0 & I_{d\times d} \end{pmatrix}, \qquad (n^{\alpha}) = \begin{pmatrix} 1\\ 0\\ 0\\ \vdots\\ 0 \end{pmatrix}, \qquad \Omega = 1$$

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One finds  $\mathscr{L}_n g_{\alpha\beta}(=-2K_{\alpha\beta})=0$ , so that metric-preserving, symmetric connections exist.

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One finds  $\mathscr{L}_n g_{\alpha\beta}(=-2K_{\alpha\beta})=0$ ,

so that metric-preserving, symmetric connections exist. (One can take  $\Gamma^{\alpha}_{\beta\gamma} = 0$ , a choice adapted to the underlying linear structure.)

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$$\delta x^0 = a^0 + b_k x^k, \quad \delta x^k = \omega_m^k x^m + a^k, \quad \omega_{km} = -\omega_{mkn}$$

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While the flat tensors  $g_{\alpha\beta}$  and  $\Omega$  are numerically invariant under Carroll transformations (by definition of the Carroll group), this is not so for the extra structure  $\theta_{\alpha}$ .

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$$(\theta_{\alpha}) = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix},$$

but this special form is not preserved in all Carroll frames.

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but in general (( $\theta_{\alpha}$ ) = (1,  $\theta_{a}$ )), it reads

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but in general  $((\theta_{\alpha}) = (1, \theta_a))$ , it reads

$$(G^{\alpha\beta}) = \begin{pmatrix} \delta^{cd}\theta_c\theta_d & -\delta^{bc}\theta_c \\ -\delta^{ac}\theta_c & \delta^{ab} \end{pmatrix}$$

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$$(G^{\alpha\beta}) = \begin{pmatrix} \delta^{cd}\theta_c\theta_d & -\delta^{bc}\theta_c \\ -\delta^{ac}\theta_c & \delta^{ab} \end{pmatrix}$$

It is thus not numerically invariant under Carroll boosts, for which one can actually verify that

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$$(G^{\alpha\beta}) = \begin{pmatrix} 0 & 0 \\ 0 & I_{d\times d} \end{pmatrix}$$

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It is thus not numerically invariant under Carroll boosts, for which one can actually verify that

$$G^{\prime\alpha\beta}\equiv\frac{\partial x^{\prime\alpha}}{\partial x^{\mu}}\frac{\partial x^{\prime\beta}}{\partial x^{\nu}}G^{\lambda\mu}=G^{\alpha\beta}(g_{\rho\sigma},n^{\tau},\theta_{\gamma}^{\prime}\equiv\theta_{\delta}\frac{\partial x^{\delta}}{\partial x^{\gamma}}).$$

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## We now consider Carroll-invariant local field theories.

A Carroll transformation is generated in the canonical formalism by

$$a^0E + a^kP_k + b_kB^k + \frac{1}{2}\omega_{km}M^{km},$$

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where the Carroll generators are given by integrals of local densities involving the "energy density"  $\mathscr{E}(x)$  and the "momentum density"  $\mathscr{P}_k(x)$ .

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The spacetime translations are generated by

$$E = \int d^d x \mathscr{E}(x), \qquad P_k = \int d^d x \mathscr{P}_k(x),$$

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The spacetime translations are generated by

$$E = \int d^d x \mathscr{E}(x), \qquad P_k = \int d^d x \mathscr{P}_k(x),$$

while the generators of Carroll boosts and spatial rotations read

$$B^k = \int d^d x x^k \mathcal{E}(x), \qquad M^{rs} = \int d^d x (x^r \delta^{sk} - x^s \delta^{rk}) \mathcal{P}_k(x).$$

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$$\begin{split} & [P_k, B^m] = \delta_k^m E, \\ & [P_k, M^{rs}] = (\delta_k^r \delta^{sl} - \delta_k^s \delta^{rl}) P_l, \qquad [B^k, M^{rs}] = -B^r \delta^{sk} + B^s \delta^{rk}, \\ & [M^{km}, M^{rs}] = -\delta^{kr} M^{ms} + \delta^{mr} M^{ks} + \delta^{ks} M^{mr} - \delta^{ms} M^{kr} \end{split}$$

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 $P_k$  and  $M_{rs}$  are kinematical generators,

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 $P_k$  and  $M_{rs}$  are kinematical generators, while *E* and  $B_k$  involve the dynamics.

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## The characteristic feature of Carroll-invariant field theories is the Poisson bracket relation

$$[\mathcal{E}^E(x), \mathcal{E}^E(y)] = 0 \qquad (\text{or } [\mathcal{E}^M(x), \mathcal{E}^M(y)] = 0)$$

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between the energy density at two different spatial points. This implies straightforwardly

$$[E, B_k] = 0$$
  $[B_k, B_m] = 0$ 

for  $E = \int d^3 x \mathscr{E}$  and  $B_k = \int d^3 x x_k \mathscr{E}$  (with  $\mathscr{E} = \mathscr{E}^E$  or  $\mathscr{E}^M$ ).

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If one also requests that  $\mathscr{E}(x)$  be a scalar under spatial isometries, the other commutation relations of the Carroll algebra are all fulfilled.

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If one also requests that  $\mathscr{E}(x)$  be a scalar under spatial isometries, the other commutation relations of the Carroll algebra are all fulfilled.

(In the Poincaré case,  $[\mathcal{E}(x),\mathcal{E}(y)]\sim (\mathcal{P}^k(x)+\mathcal{P}^k(y))\delta_{,k}(x-y)$  : Schwinger - Dirac)

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### The Hamiltonian actions of the flat space Carroll contractions can be cast in a manifestly Carroll-covariant form. We consider for definiteness electromagnetism

$$S[A_i, \pi^i, A_t] = \int dt \left[ \int d^d x \pi^a \dot{A}_a - H + \int d^d x A_t \partial_a \pi^a \right], \ H = \int d^d x \mathscr{E}^C$$

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and start with the electric case ( $\mathcal{E}^C = \mathcal{E}^E = \frac{1}{2}\pi^a \pi_a$ ).

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and start with the electric case  $(\mathscr{E}^C = \mathscr{E}^E = \frac{1}{2}\pi^a\pi_a)$ . By eliminating the momenta by means of their own field equations, one gets  $S^E[A_i, A_0] = \frac{1}{2}\int d^D x F_{0i}F_0^i$ ,

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$$S^E[A_{\alpha}] = \frac{1}{2} \int d^D x (n^{\alpha} F_{\alpha\beta})^2.$$

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$$S^{E}[A_{\alpha}] = \frac{1}{2} \int d^{D} x (n^{\alpha} F_{\alpha\beta})^{2}.$$

The integrand  $G^{\rho\sigma} n^{\alpha} F_{\alpha\rho} n^{\beta} F_{\beta\sigma}$  is well-defined because  $F_{\alpha\beta} n^{\beta}$  is transverse,  $n^{\alpha} F_{\alpha\beta} n^{\beta} = 0$ .

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One cannot express the momenta  $\pi^i$  in terms of the velocities through their equations of motion and for that reason, one looks for a direct covariantization of the first-order Hamiltonian action.

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• we postulate that the momenta  $\pi^a$  are the spatial components of a spacetime vector  $\pi^{\alpha}$  with the gauge invariance  $\pi^{\alpha} \rightarrow \pi^{\alpha} + \lambda n^{\alpha}$  ( $\lambda$  arbitrary) to keep the number of degrees of freedom unchanged;

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- (a) we introduce a 1-form gauge field  $\theta_{\alpha}$  that enables one to define  $G^{\alpha\beta}$ .

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$$S^{M}[A_{\alpha},\pi^{\beta},\theta_{\gamma}] = \int d^{D}x \left(\pi^{\alpha}F_{\alpha\beta}n^{\beta} - \frac{1}{4}G^{\alpha\beta}G^{\rho\sigma}F_{\alpha\rho}F_{\beta\sigma}\right)$$

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The gauge invariance  $\pi^{\alpha} \rightarrow \pi^{\alpha} + \lambda n^{\alpha}$  is obvious since  $F_{\alpha\beta}$  is antisymmetric -  $\pi^0$  just drops.

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Similarly, if one shifts  $\theta_{\alpha}$  as  $\delta\theta_{\alpha} = \lambda_{\alpha}$  (with  $\lambda_{\alpha}n^{\alpha} = 0$ ) and at the same time transforms  $\pi^{\alpha}$  as

$$\delta \theta_{\alpha} = \lambda_{\alpha}, \qquad \delta \pi^{\alpha} = -G^{\alpha \rho} F_{\rho \sigma} \lambda^{\sigma}$$

the action is invariant (one has  $\delta G^{\alpha\beta} = -n^{\alpha}\lambda^{\beta} - n^{\beta}\lambda^{\alpha}$ ).

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The construction can be generalized to arbitrary *p*-forms.

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# Again, the limits are most conveniently taken in the Hamiltonian formulation.

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$$\begin{split} & [\mathcal{H}^{E}(x), \mathcal{H}^{E}(x')] = 0, \\ & [\mathcal{H}^{E}(x), \mathcal{H}_{k}(x')] = (\mathcal{H}^{E}(x) + \mathcal{H}^{E}(x'))\delta_{,k}(x - x') \\ & [\mathcal{H}_{m}(x), \mathcal{H}_{k}(x')] = \mathcal{H}_{m}(x')\delta_{,k}(x - x') + \mathcal{H}_{k}(x)\delta_{,m}(x - x') \end{split}$$

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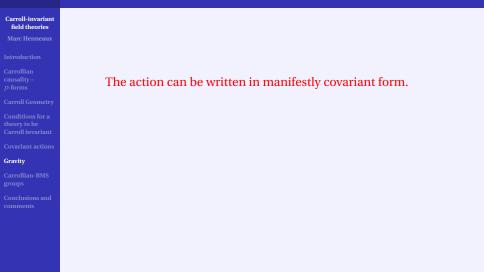
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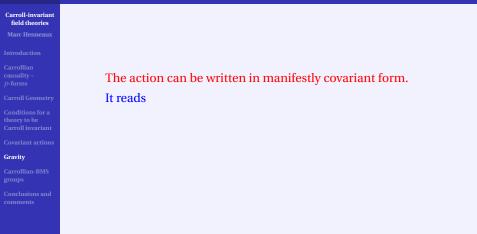
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and one sees therefore that  $c \to 0$  (Carroll limit) is equivalent to  $G \to \infty$  (strong coupling limit, Isham 1975) or  $\epsilon = 0$  (zero Hamiltonian signature limit -Teitelboim 1978), keeping  $N^{resc}$  finite.

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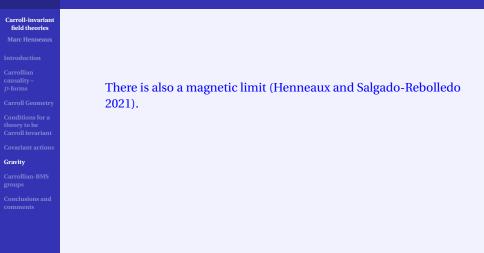
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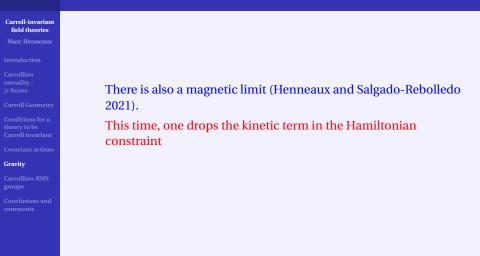
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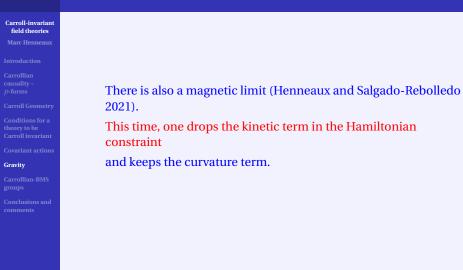
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# Carroll-invariant field theories There is also a magnetic limit (Henneaux and Salgado-Rebolledo 2021). This time, one drops the kinetic term in the Hamiltonian constraint and keeps the curvature term. Gravity This is again consistent.

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One now finds  $\dot{g}_{ii} \sim 0$ .

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*iso*(3)-BMS algebra : quotient by all even supertranslations and the boosts.

### Carroll-invariant field theories

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Conclusions and comments The subalgebra of homogeneous Carroll transformations is spanned by the spatial rotations and the boosts. It is six-dimensional and isomorphic to  $so(3) \oplus_{\sigma} t_3$ .

Carroll-BMS (*C-BMS*) algebra : The Carroll algebra can be extended by Carroll supertranslations, which form an infinite-dimensional representation of the homogeneous Carroll subalgebra and commute among themselves.

Supertranslations can be decomposed into an even part T and an odd part W.

 $d_3$ -BMS algebra : quotient by all even supertranslations (which includes the energy).

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(A. Pérez 2021 ; O. Fuentealba, M. Henneaux, Patricio Salgado-Rebolledo and J. Salzer, in preparation)

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# THANK YOU!