# Fractons on curved spacetime

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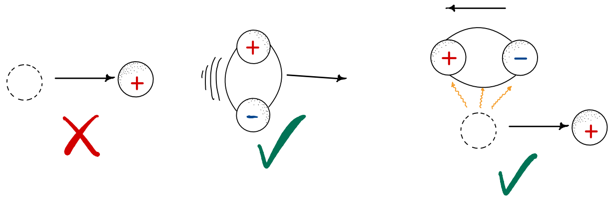
# What are fractons?

- Fracton phases of matter characterised by immobile excitations [Chamon '04; Haah '11; Pretko '17]
- Immobility can be understood as a consequence of dipole moment conservation

$$0 = \dot{\vec{d}} = q\dot{\vec{x}} \Rightarrow \dot{\vec{x}} = 0$$

- Has potential applications for self-correcting quantum memory [Haah '11]



# The plan

1. Field theories with dipole symmetry
2. Aristotelian geometry
3. Fractons on Aristotelian geometry

# Conserved dipole charge

- Consider a theory of a complex scalar $\Phi$ described by the action $S[\Phi]$
- If invariant under $\delta_\alpha \Phi = i\alpha\Phi$, Noether charge $Q_{(0)}$ given by

$$\delta_{\alpha(t)} S[\Phi] = \int dt\, \dot{\alpha}(t) Q_{(0)} = \int dt \int d^d x\, \delta_{\alpha(t)}[\text{kin. term}]$$

$$= \int dt\, \dot{\alpha}(t) \boxed{\int d^d x\, \rho}$$

- Conserved dipole charge $Q^i_{(2)} = \int d^d x\, x^i \rho$ corresponds to the global symmetry

$$\delta_\beta \Phi = i\beta_i x^i \Phi$$

# Field theories with (linear) dipole symmetry

- Dipole symmetry strongly constrains terms with spatial derivatives, e.g., $\partial_i \Phi \partial_i \Phi^\star$ *not* allowed
- Can classify Lagrangians $\mathcal{L}[\Phi, \dot{\Phi}, \partial_i \Phi, \partial_i \partial_j \Phi, \text{c.c.}]$ with the symmetry $\delta\Phi = i(\alpha + \beta_i x^i)\Phi$ amounts to

$$\delta\mathcal{L} = \left(\alpha + \beta_k x^k\right)\left[i\Phi\frac{\partial\mathcal{L}}{\partial\Phi} + i\dot{\Phi}\frac{\partial\mathcal{L}}{\partial\dot{\Phi}} + i\partial_i\Phi\frac{\partial\mathcal{L}}{\partial\partial_i\Phi} + i\partial_i\partial_j\Phi\frac{\partial\mathcal{L}}{\partial\partial_i\partial_j\Phi} + \text{c.c}\right]$$
$$+ \beta_i\left[i\Phi\frac{\partial\mathcal{L}}{\partial(\partial_i\Phi)} + 2i\partial_j\Phi\frac{\partial\mathcal{L}}{\partial(\partial_i\partial_j\Phi)} + \text{c.c}\right] = 0$$

- In particular includes the archetypical fracton Lagrangian

$$\mathcal{L} = \dot{\Phi}\dot{\Phi}^\star - m^2\left|\Phi\right|^2 - \lambda(\partial_i\Phi\partial_j\Phi - \Phi\partial_i\partial_j\Phi)(\partial_i\Phi^\star\partial_j\Phi^\star - \Phi^\star\partial_i\partial_j\Phi^\star)$$

$\Rightarrow$ *non-Gaussian*

[Pretko '18]

# Symmetry algebra

The Lagrangian

$$\mathcal{L} = \dot{\Phi}\dot{\Phi}^\star - m^2 |\Phi|^2 - \lambda(\partial_i\Phi\partial_j\Phi - \Phi\partial_i\partial_j\Phi)(\partial_i\Phi^\star\partial_j\Phi^\star - \Phi^\star\partial_i\partial_j\Phi^\star)$$

is invariant under rotations, spacetime translations and $\delta\Phi = i(\alpha + \beta_i x^i)\Phi$. The charge algebra is

$$\{M_{ij}, M_{kl}\} = -4\delta_{[k[i}M_{j]l]} \qquad \{M_{jk}, P_i\} = -2\delta_{i[j}P_{k]}$$

$$\{P_i, Q_j^{(2)}\} = \delta_{ij}Q^{(0)} \qquad \{M_{jk}, Q_i^{(2)}\} = -2\delta_{i[j}Q_{k]}^{(2)}$$

# Gauging the dipole symmetry

- We want our Lagrangian to be invariant under gauge transformations of the form $\delta_\Lambda \Phi = i\Lambda(x,t)\Phi$
- Introduce symmetric tensor gauge field and a scalar gauge field with the gauge transformations [Pretko '19]

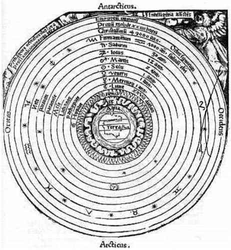
$$\delta_\Lambda A_{ij} = \partial_i \partial_j \Lambda(x,t)\,, \qquad \delta_\Lambda \phi = \dot{\Lambda}(x,t)$$

- Leads to gauge invariant Lagrangian

$$\begin{aligned}\mathcal{L} = &(\partial_t \Phi - i\phi\Phi)(\partial_t \Phi^\star + i\phi\Phi^\star) - m^2 \left|\Phi\right|^2 \\ &- \lambda(\partial_i\Phi\partial_j\Phi - \Phi\partial_i\partial_j\Phi + iA_{ij}\Phi^2)(\partial_i\Phi^\star\partial_j\Phi^\star - \Phi^\star\partial_i\partial_j\Phi^\star - iA_{ij}{\Phi^\star}^2)\end{aligned}$$

- These can be made dynamical, but will remain background fields for us

# Aristotelian geometry




- Aristostelian geometry describes abolute time and space [Penrose '68; de Boer et al. '20]
- The geometric data consists of $(\tau_\mu, v^\mu, h_{\mu\nu}, h^{\mu\nu})$, satisfying

$$v^\mu \tau_\mu = -1\,, \qquad \delta^\mu_\nu = -v^\mu \tau_\nu + h^{\mu\rho} h_{\rho\nu}$$

- Minimal "metric compatibility" of affine connection $\nabla$ with components

$$\Gamma^\rho_{\mu\nu} = -v^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\lambda} \left(\partial_\mu h_{\lambda\nu} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}\right) - h^{\rho\lambda} \tau_\nu K_{\mu\lambda}$$

- Simplifying assumption: vanishing intrinsic torsion [Figueroa-O'Farrill '20]

$$(d\tau)_{\mu\nu} = 0\,, \qquad -\tfrac{1}{2}\mathcal{L}_v h_{\mu\nu} =: K_{\mu\nu} = 0$$

# Field theory on Aristotelian backgrounds

General variation of action $S[\Phi; \tau, h_{\mu\nu}]$

$$\delta S = \int \mathrm{d}^{d+1}x\, e \left( T^{\mu} \delta\tau_{\mu} + \frac{1}{2} T^{\mu\nu} \delta h_{\mu\nu} + \mathcal{E}_{\Phi} \delta\Phi \right)$$

- Energy current ($T^{\mu}$)
- Momentum-stress tensor ($T^{\mu\nu}$)
- E-L eq. for $\Phi$ ($\mathcal{E}_{\Phi}$)

Energy-momentum tensor given by [de Boer et al. '20]

$$T^{\mu}{}_{\nu} = -T^{\mu}\tau_{\nu} + T^{\mu\rho} h_{\rho\nu}$$

Reparameterisation Ward identity tells us that

$$\nabla_{\mu} T^{\mu}{}_{\nu} + \text{torsion} = 0$$

# Complex scalar theory on curved spacetime

$$\mathcal{L} = (\partial_t \Phi - i\phi\Phi)(\partial_t \Phi^\star + i\phi\Phi^\star) - m^2 |\Phi|^2$$
$$- \lambda(\partial_i \Phi \partial_j \Phi - \Phi \partial_i \partial_j \Phi + iA_{ij}\Phi^2)(\partial_i \Phi^\star \partial_j \Phi^\star - \Phi^\star \partial_i \partial_j \Phi^\star - iA_{ij}\Phi^{\star 2})$$

- Introduce the curved space field $A_{\mu\nu}$ satisfying $v^\mu A_{\mu\nu} = 0$ and $\phi$ transforming as

$$\delta A_{\mu\nu} = P^\rho_\mu P^\sigma_\nu \nabla_\rho \partial_\sigma \Lambda\,, \qquad \delta\phi = -v^\mu \partial_\mu \Lambda$$

with $P^\mu_\nu = h^{\mu\rho} h_{\rho\nu} = \delta^\mu_\nu + v^\mu \tau_\nu$

$$\mathcal{L} = (-v^\mu \partial_\mu \Phi - i\phi\Phi)(-v^\mu \partial_\mu \Phi^\star + i\phi\Phi^\star) - m^2 |\Phi|^2$$
$$- h^{\mu\nu} h^{\rho\sigma} \lambda(\partial_\mu \Phi \partial_\nu \Phi - \Phi \partial_\mu \partial_\nu \Phi + iA_{\mu\nu}\Phi^2)(\partial_\rho \Phi^\star \partial_\sigma \Phi^\star - \Phi^\star \partial_\rho \partial_\sigma \Phi^\star - iA_{\rho\sigma}\Phi^{\star 2})$$

# Outlook

- We have shown how to couple “fractonic” theories to curved spacetime
- A derivation of the scalar field theory from a lattice description is still lacking
- Fracton fluids using geometric methods
- ...and much more!

Thank you for your attention