Holographic phase diagram of a chiral symmetry breaking gauge theory

Phase diagram of D3/D7 on T-mu with B

arXiv:1003.2694[hep-th]
N Evans, A Gebauer, KK, M Magou

Keun-young Kim
Southampton U.
1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature (T) or density (d) or chemical potential (mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
Motivation and introduction

1. Phase diagram for strong coupling regime by holography.
   (For example, chiral symmetry breaking in QCD)

   (Theoretical, AdS/CMT)

3. Two examples (probe limit)
   1) D4/D8: Non-Abelian flavor chiral symmetry
      (S. Sugimoto yesterday)
   2) D3/D7: $U(1)_R$ symmetry $\sim U(1)$ axial symmetry
      - quark mass and condensate in a natural way
      - similar to AdS/CMT superconductor story (D3/D5)
Outline

1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature(T) or density(d) or chemical potential(mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
Holographic thermodynamics 1

**Equilibrium**

- Field-operator correspondence
- BH thermodynamics

Holography

- On-shell Euclidean action with the bulk fields representing thermodynamic variables

**Non-equilibrium**

(Reviewed by R. Myers and A. Buchel)

- ex) Transport coefficient $\sim$ Kubo formula $\sim$ Retarded Green’s function

(Reviewed by R. Myers and A. Buchel)
Holographic thermodynamics

\[ Z = \langle e^{-\int d^4 x \mathcal{O}\Phi_0} \rangle_{FT} = e^{-\int d^5 x \mathcal{L}_{\text{gravity}}[\Phi \to \Phi_0]} \]

\[ Z = e^{-\beta F} \]

4D Field theory

Partition function

5D Gravity

Partition function

\[ e^{-\int d\tau \int d\vec{x} \int dz \mathcal{L}_{\text{gravity}}[\Phi(z;T,\mu,B,m,\cdots)]} = e^{-\beta V} \int dz \mathcal{L}_{\text{gravity}}[\Phi(z;T,\mu,B,m,\cdots)] = e^{-\beta V} \frac{F}{V} \]

\( g_{\mu\nu} : g_{tt} \sim 1 - \frac{zH}{z} \)

\( A_\mu : A_0 \to \mu \)

\( A_{\mu}^{\text{ext}} : A_{y}^{\text{ext}} = Bx \)

\( \phi : L \to m \)
Outline

1. Motivation and main result
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature(T) or density(d) or chemical potential(mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
<table>
<thead>
<tr>
<th>Classical strings</th>
<th>'t Hooft limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_s \to 0$, $R/l_s$ fixed.</td>
<td>$N \to \infty$, $\lambda = g_{YM}^2 N_c = \lambda$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Classical supergravity</th>
<th>Large 't Hooft coupling limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_s \to 0$, $R/l_s \to \infty$.</td>
<td>$N \to \infty$, $\lambda \to \infty$</td>
</tr>
</tbody>
</table>
D3/D7 model (probe limit)

**Deformation**

1. Fundamental degrees of freedom:
   - Adding Nf flavor branes (Nc >> Nf) in probe limit
   - Flavor dynamics: Effective D brane action (DBI+WZ) action in the fixed background
2. Temperature: AdS$_5$ black hole
3. Flavor chemical potential (density): Gauge field, At on the flavor brane

![Fig. from Edelstein et al. (09)](image)

**Flavor dynamics**

\[ S_{DBI} \sim \int d^{p+1}x e^{-\phi} \sqrt{-\det(P g_{MN} + 2\pi\alpha' F_{MN})} \]

\[ S_{CS} \sim \int C \wedge \text{tr} e^{2\pi\alpha' F} \]

\[ S_{DBI} \sim e^{-\phi} g \cdot F \]  (Dirac-Bron-Infeld)
Field theory and its gravity dual

N=2 SUSY Gauge theory: N=4 SU(Nc) SUPER YM + N=2 Nf Fundamental Hypermultiplet

\[ \mathcal{L} = \text{Im} \left[ \tau \int d^4 \theta \left( \text{tr} (\Phi_I e^V \Phi_I e^{-V}) + Q^i_r e^V Q^i_r + \bar{Q}^i_r e^{-V} \bar{Q}^i_r \right) + \tau \int d^2 \theta (\text{tr} (W^a W^a) + W) + c.c. \right], \quad W = \text{tr} \left( \varepsilon_{IJK} \Phi_I \Phi_J \Phi_K \right) + \bar{Q}_r (m_q + \Phi_3) Q^r \]

<table>
<thead>
<tr>
<th>\mathcal{N} = 2</th>
<th>components</th>
<th>spin</th>
<th>SU(2)_Φ × SU(2)_R</th>
<th>U(1)_R</th>
<th>Δ</th>
<th>U(N_f)</th>
<th>U(1)_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Φ_1, Φ_2) hyper</td>
<td>X^4, X^5, X^6, X^7</td>
<td>0 ( \frac{1}{2} )</td>
<td>( \left( \frac{1}{2}, \frac{1}{2} \right) )</td>
<td>0</td>
<td>1</td>
<td>( \frac{3}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>(Φ_3, W_α) vector</td>
<td>X^A_V = (X^8, X^9)</td>
<td>0 ( \frac{1}{2} )</td>
<td>( 0, \frac{1}{2} )</td>
<td>+2</td>
<td>1</td>
<td>( \frac{3}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>(Q, \bar{Q}) fund. hyper</td>
<td>( q^m = (q, \bar{q}) )</td>
<td>0 ( \frac{1}{2} )</td>
<td>( 0, \frac{1}{2} )</td>
<td>0</td>
<td>1</td>
<td>( \frac{3}{2} )</td>
<td>( N_f )</td>
</tr>
<tr>
<td></td>
<td>( \psi_i = (\psi, \bar{\psi}^i) )</td>
<td>( \frac{1}{2} )</td>
<td>( 0, 0 )</td>
<td>( \mp 1 )</td>
<td>1</td>
<td>( \frac{3}{2} )</td>
<td>( N_f )</td>
</tr>
</tbody>
</table>

Nc D3 branes + Nf D7 branes

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>D3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Boundary action: \( m_q \tilde{Q} Q \sim m_q (\bar{\psi} \psi + \cdots) \)

Embedding bulk field: \( L(\rho) \sim m_q + \frac{<\tilde{Q} Q>}{\rho^2} \)

R-symmetry \sim \) Rotation symmetry
\( (X_4 \sim X_9) \) \( (X_4 \sim X_7) \) \( (X_8 \sim X_9) \)
\( SO(6)_R \rightarrow SO(4) \times SO(2) \)
\( \sim SU(2)_\Phi \times SU(2)_R \times U(1)_R \) \( \sim U(1)_A \)

Title of this talk: Holographic Phase diagram of “chiral symmetry” breaking gauge theory
D3/D7 set up at zero T

\[
\begin{array}{cccccccccc}
X_0 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 \\
D3 & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
D7 & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
\end{array}
\]

\[ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2) + \frac{R^2}{r^2}d\rho^2 + R^2d\Omega_5^2\]

\[r = \sqrt{\rho^2 + L^2}, \quad \rho := r \sin \theta, \quad L := r \cos \theta\]

\[ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2) + \frac{R^2}{r^2}(d\rho^2 + \rho^2d\Omega_3^2 + dL^2 + L^2d\Omega_1^2)\]

Embedding function: \( L(\rho) \)

\( L(X_0 \sim X_7), \Omega_1(X_0 \sim X_7) \)
DBI action and solution

Background metric

\[ ds^2 = \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\Omega_1^2) \]

DBI action

\[ S_{DBI} = -T_{D7} \int d^8 \xi \sqrt{-\text{det}(P[G]_{ab} + 2\pi \alpha' F_{ab})} \]

\[ P[G]_{ab} = \begin{pmatrix}
-\frac{r^2}{R^2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{r^2}{R^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{r^2}{R^2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{r^2}{R^2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{R^2}{r^2} (1 + L^2) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{R^2}{r^2} \Omega_3 \\
\end{pmatrix} \]

\[ S = \int d\xi^8 \mathcal{L}(\rho) = \left( \int_{S^3} \epsilon_3 \int dt d\vec{x} \right) \int d\rho \mathcal{L}(\rho) \]

1D effective action

\[ \mathcal{L} := -N_f T_{D7} \frac{\rho^3}{4} \sqrt{1 + (\partial_\rho L)^2} \]

E.O.M. & Solution

\[ \partial_\rho \left( \rho^3 \frac{L'}{\sqrt{1 + L'^2}} \right) = 0 \quad \Rightarrow \quad L(\rho) \sim C_1 + C_2 \frac{1}{\rho^2} \]

\[ m \text{ (quark mass)} \quad c \text{ (condensate)} \]
\[ \partial_\rho \left( \rho^3 \frac{L'}{\sqrt{1 + L'^2}} \right) = 0 \quad \implies \quad L(\rho) = \text{constant} \]
1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature (T) or density (d) or chemical potential (mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
**Action with magnetic field**

**Background metric**

\[
d s^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\Omega_1^2)
\]

**DBI action**

\[
S_{DBI} = -T_{D7} \int d^8 \xi \sqrt{-\det(P[G]_{ab} + 2\pi \alpha' F_{ab})}
\]

\[
P[G]_{ab} + 2\pi \alpha' F_{ab} = \begin{pmatrix}
-\frac{r^2}{R^2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{r^2}{R^2} & B & 0 & 0 & 0 \\
0 & -B & \frac{r^2}{R^2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{r^2}{R^2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{R^2}{r^2} (1 + L^2) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{R^2}{r^2} \Omega_3
\end{pmatrix}
\]

\[
S = \int d\xi^8 \mathcal{L}(\rho) = \left( \int_{S^3} \epsilon_3 \int dt d\vec{x} \right) \int d\rho \mathcal{L}(\rho)
\]

\[
\mathcal{L} = -N_f T_{D7} \frac{\rho^3}{4} \sqrt{1 + (\partial_\rho L)^2} \sqrt{1 + \frac{4R^4}{(\rho^2 + L^2)^2} B^2}
\]

\[
= -N_f T_{D7} (R \sqrt{B})^4 \frac{\tilde{\rho}^3}{4} \sqrt{1 + (\partial_\tilde{\rho} \tilde{L})^2} \sqrt{1 + \frac{1}{(\tilde{\rho}^2 + \tilde{L}^2)^2}}
\]

\[
(\tilde{w}, \tilde{L}, \tilde{\rho}) := \left( \frac{w}{R \sqrt{2B}}, \frac{L}{R \sqrt{2B}}, \frac{\rho}{R \sqrt{2B}} \right)
\]
Curved embedding and condensate

Embedding L vs rho

- c vs m

Embedding at m = 0

Finite slope at infinity
= Finite condensate
= Chiral symmetry broken

Repulsion effect by B
~ Dynamical quark mass

Lowest energy

UV Input
IR regularity

\[
\tilde{L}(\tilde{\rho}) \sim \tilde{m} + \frac{\tilde{c}}{\tilde{\rho}^2}
\]

\[
\tilde{L}'(\tilde{\rho}) \sim -\frac{\tilde{c}^2}{\tilde{\rho}^3}
\]
1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature (T) or density (d) or chemical potential (mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
II. THE HOLOGRAPHIC DESCRIPTION

The existence of two critical points is related with the existence of two critical points in 

The magnetic field case provides a smooth transition which is first order at large temperature. The infinite mass limit corresponds to the pure DBI action we will consider is then 

and we can use the Legendre transformation to introduce a constant magnetic field (eg . The full DBI action we will consider is then 

where is the position of the black hole horizon which becomes a cross over the moment a mass is introduced. The first order transition structure though remains, even to the magnetic field. The infinite mass limit corresponds to the pure DBI action.

The holographic description in terms of an AdS geometry is the cleanest known example of chiral symmetry breaking in a holographic environment. Other configurations break chiral symmetry but apparently second order at low temperature.

Other configurations break chiral symmetry but apparently second order at low temperature.

We embed the D7 brane in the AdS \( \text{d}S^4 \) at constant \( \rho \) and we can use the Legendre transformation to introduce a constant magnetic field (eg . The full DBI action we will consider is then 

where is the gauge field living on the D7 world volume. We will use the notation \( w \) for the radial coordinate of the D7 brane. The magnetic field case provides a smooth transition which is first order at large temperature. The infinite mass limit corresponds to the pure DBI action we will consider is then 

where is the position of the black hole horizon which becomes a cross over the moment a mass is introduced. The first order transition structure though remains, even to the magnetic field. The infinite mass limit corresponds to the pure DBI action.

The existence of two critical points is related with the existence of two critical points in 

The magnetic field case provides a smooth transition which is first order at large temperature. The infinite mass limit corresponds to the pure DBI action we will consider is then 

where is the position of the black hole horizon which becomes a cross over the moment a mass is introduced. The first order transition structure though remains, even to the magnetic field. The infinite mass limit corresponds to the pure DBI action.
Adding chemical potential (density)

**DBI action**

\[ S_{DBI} = -T_{D7} \int d^8 \xi \sqrt{-\det(P[G]_{ab} + 2\pi \alpha' F_{ab})} \]

\[ P[G]_{ab} + 2\pi \alpha' F_{ab} = \begin{pmatrix}
- g_t \frac{w^2}{R^2} & 0 & 0 & 0 & - A'_t & 0 \\
0 & g_x \frac{w^2}{R^2} & B & 0 & 0 & 0 \\
0 & -B & g_x \frac{w^2}{R^2} & 0 & 0 & 0 \\
0 & 0 & 0 & g_x \frac{w^2}{R^2} & 0 & 0 \\
A'_t & 0 & 0 & 0 & \frac{R^2}{w^2} (1 + L'^2) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{R^2}{w^2} \Omega_3
\end{pmatrix} \]

\[ S = \int d^8 \xi \mathcal{L}(\rho) = \left( \int_{S^3} \epsilon_3 \int dtd\bar{x} \right) \int d\rho \mathcal{L}(\rho) \]

**1D effective action**

\[ \mathcal{L} = -N_f T_{D7} \rho^3 \left( 1 - \frac{w^4_H}{w^4} \right) \sqrt{\left( 1 + (\partial_\rho L)^2 - \frac{2w^4(w^4 + w^4_H)}{(w^4 - w^4_H)^2}(2\pi \alpha' \partial_\rho A_t)^2 \right)} \left( \left( 1 + \frac{w^4_H}{w^4} \right)^2 + \frac{4R^4}{w^4} B^2 \right) \]

**Legendre transformation**

\[ \tilde{S} = S - \int d^8 F_{\rho t} \frac{\delta S}{\delta F_{\rho t}} = \left( \int_{S^3} \epsilon_3 \int dtd\bar{x} \right) \int d\rho \tilde{\mathcal{L}}(\rho) \]

\[ d := \frac{\delta S}{\delta F_{\rho t}} \]

\[ \tilde{\mathcal{L}} = -N_f T_{D7} (R\sqrt{B})^4 \frac{\tilde{w}^4 - \tilde{w}^4_H}{\tilde{w}^4} \sqrt{\left( 1 + \tilde{L}'^2 \right)} \left[ \left( \frac{\tilde{w}^4 + \tilde{w}^4_H}{\tilde{w}^4} \right)^2 \tilde{\rho}^6 + \frac{1}{\tilde{w}^4} \tilde{\rho}^6 + \frac{\tilde{w}^4}{(\tilde{w}^4 + \tilde{w}^4_H)^2} \tilde{d}^2 \right] \]
Holographic thermodynamic functions

**Hamilton equation**

\[ \partial_\rho d = \frac{\delta S}{\delta A_t} \]

**Charge conservation**

\[ \partial_\rho A_t = -\frac{\delta S}{\delta d} \quad \implies \quad \partial_\rho \tilde{A}_t = \tilde{d} \frac{\tilde{w}_4^4 - \tilde{w}_H^4}{\tilde{w}_4^4 + \tilde{w}_H^4} \sqrt{\frac{1 + (\tilde{L}')^2}{\tilde{K}}} \]

\[ \tilde{A}_t \approx \tilde{\mu} - \frac{\tilde{d}}{2\tilde{\rho}^2} \]

**Chemical potential**

\[ \tilde{\mu} := \lim_{\tilde{\rho} \to \infty} \tilde{A}_t(\tilde{\rho}) = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \tilde{d} \frac{\tilde{w}_4^4 - \tilde{w}_H^4}{\tilde{w}_H^4} \sqrt{\frac{1 + (\tilde{L}')^2}{\tilde{K}}} \]

**Grand potential**

\[ \tilde{\Omega}(\tilde{w}_H, \tilde{\mu}, \tilde{m}) := \frac{-S}{N_f T_D^4 (R \sqrt{B})^4 \text{Vol}} = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \frac{\tilde{w}_4^4 - \tilde{w}_H^4}{\tilde{w}_4^4} \sqrt{\frac{1 + (\tilde{L}')^2}{\tilde{K}}} \left( \left( \frac{\tilde{w}_4^4 + \tilde{w}_H^4}{\tilde{w}_4^4} \right)^2 \tilde{\rho}^6 + \frac{\tilde{w}_4^2}{\tilde{w}_4^6} \right) \]

**Helmholtz free energy**

\[ \tilde{F}(\tilde{w}_H, \tilde{d}, \tilde{m}) := \frac{-\tilde{S}}{N_f T_D^4 (R \sqrt{B})^4 \text{Vol}} = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \frac{\tilde{w}_4^4 - \tilde{w}_H^4}{\tilde{w}_4^4} \sqrt{K(1 + (\tilde{L}')^2)} \]
Symmetry restoration by T or d

Expectation from Field theory: Chiral symmetry is **restored** at high T or density

In terms of gravity language: The embedding becomes **flat** at high T or density

= There are **attractive** effect by T or density

**Symmetry restoration by Temperature**

Albash, Filev, Johnson, Kundu (07)
Erdmenger, Meyer, Shock (07)

AdS - Black hole background
Black hole horizon ~ Field theory T

**Symmetry restoration by density**

Evans, Gebaur, KK, Magou (10)
Jensen, Karch, Thompson (10)

Background gauge field At(rho)

\[ A_t(\rho) = \mu + \frac{d}{\rho^2} \]
1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature (T) or density (d) or chemical potential (mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
Phase diagram in T-mu plane (1)

Chiral symmetry restored: 1st order

Density creation: 2nd order

Chiral symmetry restored: 2nd order

Evans, Gebaur, KK, Magou (10)
Phase diagram in T-μu plane (2)

Evans, Gebaur, KK, Magou (10)
**Phase diagram in T-mu plane (3)**

As the density increases the value of the condensate for the $\tilde{\rho} = 0$ embeddings falls - we show a sequence of plots for growing $\tilde{d}$ in Fig 2b-d (Middle). There is a critical value of $\tilde{d} = 0.3197$ where $\tilde{c}$ becomes zero for the massless embeddings - above this value of $\tilde{d}$, D7 embedding is flat and lies along the $\tilde{\rho}$ axis (2c-d (Left)). One can see from the plots that there is a second order phase transition to a phase with no chiral condensate. In Fig 2c (Left) and 2d (Left) Phase diagram in $T$-$\mu$ plane (3) Evans,Gebaur,KK,Magou (10)
Symmetry restoration by chemical potential (Grand Canonical Ensemble)

\[ \tilde{\Omega}(\tilde{\omega}_H = 0, \tilde{\mu}, \tilde{m} = 0) \]

- \[ -\tilde{c} \sim - (\tilde{\mu}_c - \tilde{\mu})^{1/2} \]

First example showing continuous transition at finite density
1. Motivation and introduction
2. Holographic thermodynamics
4. Chiral symmetry breaking by magnetic field
5. Chiral symmetry restoration by Temperature (T)
   or density (d) or chemical potential (mu)
6. Phase Diagram in T-mu plane
7. Extension and other models
\[ds^2 = H^{-1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} \sum_{j=0}^{3} dx_j^2 + H^{1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2\]

\[H = \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1, \quad w^2 = \sum_{i=1}^{6} w_i^2 \quad \delta = \frac{R^4}{2b^4}\]
The dashed U-shaped curve represents a profile in vanishing background field and the solid U-shaped curve represents a profile when a non-zero magnetic field strength is present. These profiles are obtained by numerically solving the equation of motion for the probe brane. The straight line at the boundary becomes higher and higher for increasing magnetic field, but for high enough field it vanishes as the flavour branes at the boundary tend to not sense any further increase in the number of constituent mass. This corresponds to a self-consistent identification of a quasi-free state.

The joining of the flavour branes inside the core can be associated with a change in magnetisation. In other words it promoted the spontaneous breaking of chiral symmetry. This fits with the general expectations from field theory (see e.g. refs. [27]).

In figure 5(a) we see that the asymptotic separation decreases as the temperature increases. In other words it promoted the spontaneous breaking of chiral symmetry. These profiles are obtained by numerically solving the equation of motion for the probe brane. We can extract some more information about the transition by introducing an order parameter $\chi$. When the curved solutions are the lowest energy solutions (transition is accompanied by entropy density that jumps at $\Delta z \sim 0$ limit. Again with the change to variable $\frac{y}{L} \sim 0$, the dependence is more complicated and the joining of the flavour branes inside the core can be associated with the breaking of chiral symmetry. This corresponds to a self-consistent identification of a quasi-free state.

In figure 5(b) shows a similar behaviour as the low temperature case. However true only in the presence of a magnetic field. This fits with the general expectations from field theory (see e.g., ref. [27]).

$\chi$ is the asymptotic separation between the branes. We have set $y = 0$, $H = 1$ limit. The general dependence is more complicated and the joining of the flavour branes inside the core can be associated with the breaking of chiral symmetry. This corresponds to a self-consistent identification of a quasi-free state.

For any other value of $y$, $H$, for example $y = 0.3$, $H = 0.2$, $\chi$ shows an increment (therefore a change in magnetisation) whereas for small $y$, $H$ shows an increment (therefore a change in magnetisation).

In figure 8, we see in that the asymptotic separation decreases as the temperature increases. In other words it promoted the spontaneous breaking of chiral symmetry. These profiles are obtained by numerically solving the equation of motion for the probe brane. We can extract some more information about the transition by introducing an order parameter $\chi$. When the curved solutions are the lowest energy solutions (transition is accompanied by entropy density that jumps at $\Delta z \sim 0$ limit. Again with the change to variable $\frac{y}{L} \sim 0$, the dependence is more complicated and the joining of the flavour branes inside the core can be associated with the breaking of chiral symmetry. This corresponds to a self-consistent identification of a quasi-free state.

We can extract some more information about the transition by introducing an order parameter $\chi$. When the curved solutions are the lowest energy solutions (transition is accompanied by entropy density that jumps at $\Delta z \sim 0$ limit. Again with the change to variable $\frac{y}{L} \sim 0$, the dependence is more complicated and the joining of the flavour branes inside the core can be associated with the breaking of chiral symmetry. This corresponds to a self-consistent identification of a quasi-free state.
Summary

1. Model:
   N=2 gauge theory (N=4 SU(Nc) SYM + N=2 Nf fundamental Hypermultiplet
   Nc D3 branes + Nf D7 branes (Nc >> Nf)

2. Chiral symmetry considered: U(1)A ~ U(1)R
   Rotation symmetry in X8 and X9

3. Magnetic field: Repulsion effect --> Symmetry breaking

4. Temperature and density: Attractive effect: --> Symmetry restoration
   Temperature: 1st order
   Density: 2nd order

5. Competing between attraction and repulsion in T & mu
   --> Critical points and Phase diagram

6. 2nd order transition: Density effect

7. Electric field: Insulator/conductor transition

8. D4/D8: Chiral magnetic effect, Chiral magnetic spiral.
   (Ho-ung Yee, Next Monday)
Condensate near critical point at T=0

BKT: Berezinskii-Kosterlitz-Thouless
Order parameter $\sim e^{-\frac{\alpha}{\sqrt{T_c-T}}}$

$-\tilde{C} \sim e^{-\frac{\alpha}{\sqrt{d_c-d'}}}$

(a) $\tilde{w}_H = 0.15$
(b) $\tilde{w}_H = 10^{-5}$
(c) $\tilde{w}_H = 10^{-5}$ near the critical point
Phase diagram in T-μ plane at finite mass

Density effect weaken the order of phase transition