This is an experimental talk

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Talk inspired by the movie ‘Memento’

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Thanks for your attention!

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$$\Gamma[h,\, g] = \kappa \int\limits_0^\beta \mathrm{d}\tau \left(\dot{h}^2 - \dot{g}\left(\frac{2\pi i}{\beta}\,\dot{h} + \frac{\ddot{h}}{\dot{h}}\right) + \ddot{g}\right)$$

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- Q3: What is the twisted warped analogue of the Virasoro and sl(2) symmetries governing the Schwarzian?
- A3: The twisted warped symmetries

$$[L_n,\,L_m]=(n-m)\,L_{n+m}$$
$$[L_n,\,J_m]=-m\,J_{n+m}-i\kappa\,(n^2-n)\,\delta_{n+m,\,0}$$
$$[J_n,\,J_m]=0$$

and the two-dimensional Maxwell symmetries (L_1,L_0,J_{-1},J_0)

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- A3: The twisted warped and two-dimensional Maxwell symmetries
- Q4: What is the flat space analogue of SYK?

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Concrete model for flat space holography

Scaling limit of complex SYK

- Effective action of collective low temperature modes

$$\Gamma^{\text{cSYK}}[h,\,g] = \frac{NK}{2}\int\limits_0^\beta \mathrm{d}\tau\,\big(\dot g + \tfrac{2\pi i\mathcal{E}}{\beta}\,\dot h\big)^2 - \frac{N\gamma}{4\pi^2}\int\limits_0^\beta \mathrm{d}\tau\,\{\tan(\tfrac{\pi}{\beta}h);\,\tau\}$$

with Schwarzian derivative

$$\{f;\,\tau\} := \frac{\dddot f}{\dot f} - \frac{3}{2}\,\frac{\ddot f^2}{\dot f^2}$$

Definitions:
  - N: (large) number of complex fermions
  - NK: zero-temperature charge compressibility
  - $N\gamma$: specific heat at fixed charge
  - $\mathcal{E}$: spectral asymmetry parameter
  - β: inverse temperature
  - $h(\tau)$: time-reparametrization field, quasi-periodic $h(\tau+\beta) = h(\tau) + \beta$
  - $g(\tau)$: phase field

Davison, Fu, Georges, Gu, Jensen, Sachdev '16; Gu, Kitaev, Sachdev, Tarnopolsky '19

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yields 'twisted warped Schwarzian action' Afshar '19

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Coincides with boundary action obtained from CGHS

Hamiltonian formulation of twisted warped action and thermodynamics

- First order form involves three canonical pairs $(i = 1, 2, 3)$

$$\Gamma[q_i,\, p_i] = -\kappa \int_0^\beta \mathrm{d}\tau \, \big(p_i \dot{q}_i - p_1 p_2 - e^{q_1} p_3\big)$$

Note: relation to h and g as follows:

$$q_3(\tau) = e^{2\pi i h(\tau)/\beta} \qquad\qquad q_2(\tau) = g(\tau) - \tfrac{\beta}{2\pi i}\, h(\tau)$$

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- Solutions to Hamilton EOM depend on six integration constants

$$q_3 = h_0 + h_1e^{i\tau/\tau_0} \qquad\qquad q_2 = g_0 - ig_1\tau + g_2e^{i\tau/\tau_0}$$

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- inverse specific heat at fixed charge vanishes since $\mathrm{d}T/\,\mathrm{d}S = 0$

Derivation of boundary action

Follow derivation of Schwarzian action for JT in BF-formulation González, DG, Salzer '18

- Well-defined variational principle requires boundary term

$$\Gamma^{\text{BF}}[B,\,\mathcal{A}] = I^{\text{BF}}[B,\,\mathcal{A}] + I^{\text{bdry}}[B,\,\mathcal{A}]$$

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First order formulation of CGHS

- first order formulation as BF action

$$I^{\mathrm{BF}}[B,\,\mathcal{A}] = \kappa\int\langle B,\,F\rangle \qquad\qquad F = \mathrm{d}\mathcal{A} + \mathcal{A}\wedge\mathcal{A}$$

with Maxwell-algebra valued connection 1-form

$$\mathcal{A} = \omega\,J + e^a\,P_a + A\,Z$$

with non-zero commutators $[P_+,\,P_-] = Z$ and $[P_\pm,\,J] = \pm P_\pm$

interpretation of connection components:

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- bc's for connection and co-adjoint scalar

$$\mathcal{A} = b^{-1}(\mathrm{d}{+}a)\,b \qquad\qquad B = b^{-1}xb$$

with $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)\,P_+ + P_- + \mathcal{P}(u)\,J\big)\,\mathrm{d}u$$
$$x = \big(\dot{x}_0(u) + \mathcal{T}(u)x_1(u)\big)\,P_+ + x_1(u)\,P_- + Y\,J + x_0(u)\,Z$$

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- reminiscent of Chern–Simons formulation of 3d gravity

Boundary conditions in metric formulation and asymptotic Killing vectors

- in EF gauge most general solution to EOM

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- 2d Coulomb connection $A = r\,\mathrm{d}u$ preserved by

$$\delta_{\xi,\sigma}A_\nu = \xi^\mu\partial_\mu A_\nu + A_\mu\partial_\nu\xi^\mu + \partial_\nu\sigma \qquad\qquad \dot\sigma = \eta$$

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- asymptotic symmetry algebra is warped Witt, $[J_n,\,J_m]_{\text{Lie}} = 0$ and

$$[L_n,\,L_m]_{\text{Lie}} = (n-m)\,L_{n+m} \qquad\qquad [L_n,\,J_m]_{\text{Lie}} = -m\,J_{n+m}$$

Boundary conditions in metric formulation and asymptotic Killing vectors

- in EF gauge most general solution to EOM

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\big(\mathcal{P}(u)\,r + \mathcal{T}(u)\big)\;\mathrm{d}u^2$$

- bc's: allow fluctuations $\delta\mathcal{P} \neq 0 \neq \delta\mathcal{T}$
- bc's and gauge fixing preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\,\partial_u - \big(\dot\epsilon(u)r + \eta(u)\big)\,\partial_r$$

- 2d Coulomb connection $A = r\;\mathrm{d}u$ preserved by

$$\delta_{\xi,\sigma}A_\nu = \xi^\mu\partial_\mu A_\nu + A_\mu\partial_\nu\xi^\mu + \partial_\nu\sigma \qquad\qquad \dot\sigma = \eta$$

- define Laurent modes $L_n := \xi(\epsilon = -u^{n+1},\,0)$, $J_n := \xi(0,\,\sigma = u^n)$
- asymptotic symmetry algebra is warped Witt, $[J_n,\,J_m]_{\text{Lie}} = 0$ and

$$[L_n,\,L_m]_{\text{Lie}} = (n-m)\,L_{n+m} \qquad\qquad [L_n,\,J_m]_{\text{Lie}} = -m\,J_{n+m}$$

- dilaton linear in radial coordinate $X = x_1(u)\,r + x_0(u)$

CGHS-inspired action á la Cangemi–Jackiw

- Consider dilaton-Maxwell action in two dimensions

$$I_{\text{CGHS}} = \frac{\kappa}{2} \int \mathrm{d}^2x\sqrt{-g}\,\big(XR - 2Y + 2Y\varepsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

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historic note:

integrating out auxiliary field Y and Maxwell field A_μ yields geometric part of action by Callan, Giddings, Harvey, Strominger '91, see Cangemi, Jackiw '92

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  - auxiliary field Y (on-shell: $U(1)$ charge)
  - Maxwell field A_μ (on-shell: constant electric field, $A = r\ \mathrm{d}u$)
- EOM

$$R = 0$$
$$\varepsilon^{\mu\nu}\partial_\mu A_\nu = 1$$
$$\nabla_\mu \nabla_\nu X - g_{\mu\nu}\nabla^2 X = g_{\mu\nu} Y$$
$$Y = \Lambda = \text{const.}$$

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Metric locally Ricci flat ⇒ candidate for flat space holography!

Brief summary of SYK (Kitaev '15; Maldacena, Stanford '16)

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- Schwarzian action also follows from JT gravity

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- what is flat space analogue of Schwarzian action?

Flat space holography and complex SYK

Based on 1911.05739

Collaborators:

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Bonus slide on Maxwell algebra

Commutators not displayed vanish

- Maxwell algebra = centrally extended Poincaré

$$[\mathcal{P}_a,\,\mathcal{P}_b] = \epsilon_{ab}\,\mathcal{Z} \qquad\qquad [\mathcal{P}_a,\,\mathcal{J}] = \epsilon_a{}^b\,\mathcal{P}_b$$

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- Comment 1: maximal subalgebra of warped Witt

  Warped Witt ($n, m \in \mathbb{Z}$):

$$[L_n,\, L_m] = (n-m)\, L_{n+m} \qquad\qquad [L_n,\, J_m] = -m\, J_{n+m}$$

  Central extension:

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\left(n^3 - n\right) \delta_{n+m,\,0}$$

$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\left(n^2 - n\right) \delta_{n+m,\,0}$$

$$[J_n,\, J_m] = \frac{\hat{K}}{2}\, n\, \delta_{n+m,\,0}$$

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- Comment 2: contraction of $sl(2) \oplus u(1)$

Explicitly: take limit $\epsilon \to 0$ of

$$\hat{L}_\pm = \frac{1}{\epsilon}\,P_\pm \qquad \hat{L}_0 = J + \frac{1}{2\epsilon^2}\,Z \qquad \hat{J}_0 = Z$$

where $[\hat{L}_n,\,\hat{L}_m] = (n-m)\,\hat{L}_{n+m}$ and $[\hat{J}_0,\,\hat{L}_n] = 0$

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- Comment 2: contraction of $sl(2) \oplus u(1)$
- Change of basis 3: $a = L_1$, $a^\dagger = J_{-1}$, $H = \frac{1}{\hbar}\, a^\dagger a = L_0$, $\hbar\, \mathbb{1} = J_0$

$$[a^\dagger,\, a] = \hbar\, \mathbb{1} \qquad\qquad [H,\, a] = -a \qquad\qquad [H,\, a^\dagger] = a^\dagger$$

harmonic oscillator!