Towards flat space higher spin models in 2d

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Higher Spin Gravity: Chaotic, Conformal and Algebraic Aspects
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Outline

Flat space higher spin gravity in 3d

AdS higher spin gravity in 2d

Flat space spin-2 gravity in 2d

Towards flat space higher spin gravity in 2d

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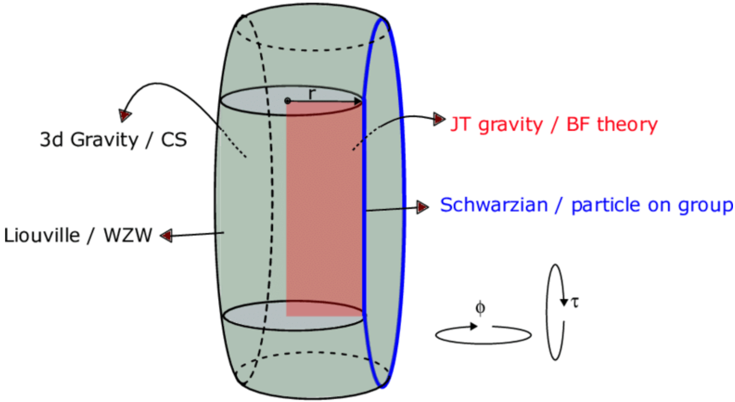
- Accessibility — we believe we can construct it
- SYK-Holography — flat space version of Schwarzian action?



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Lightning review of AdS$_3$ contraction to flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13; González, Matulich, Pino, Troncoso '13

Here is the recipe:

- Take Chern–Simons on cylinder gauge algebra contains sl$(2,\mathbb{R})\oplus$ sl$(2,\mathbb{R})$

$$I_{\text{CS}}[A] = \frac{k}{4\pi}\int_{\mathbb{R}\times\text{disk}}\langle A\wedge A + \tfrac{2}{3}\,A\wedge A\wedge A\rangle$$

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$$I_{\rm CS}[A] = I_{\rm CS}[A^+] - I_{\rm CS}[A^-]$$

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- Add bc's in each sector

$$A^{\pm} = (b^{\pm})^{-1}\big(\mathrm{d}+a^{\pm}\big)b^{\pm} \qquad \delta a^{\pm} \sim \delta\mathcal{W}^{\pm}$$

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- Stir well and get AS generators $W_n^\pm$

$$W_n^\pm = \oint_{S^1} \mathrm{d}\varphi \, e^{in\varphi} \mathcal{W}^\pm$$

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- Boil down to AS algebra

$$[W_n^\pm,\, W_m^\pm] = f(n,m)\, W_{n+m}^\pm + Z(n,m)\, \delta_{n+m}$$

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- Cook up IW contraction ($\ell =$ AdS-radius)

$$W_n := W_n^+ - W_{-n}^- \qquad \text{even}$$

$$V_n := \frac{1}{\ell}\left(W_n^+ + W_{-n}^-\right) \qquad \text{odd}$$

IW contraction: limit $\ell \to \infty$ after evaluating brackets

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- Cook up IW contraction
- Enjoy flat space AS algebra!

$$[\text{even}, \text{even}] = \text{even}$$
$$[\text{even}, \text{odd}] = \text{odd}$$
$$[\text{odd}, \text{odd}] = 0 \qquad \text{HS-supertranslations}$$

HS generalization of BMS$_3$ (a.k.a. BMW)

Simplest example: flat space spin-3 gravity

- Take spin-3 gravity (sl(3) with principally embedded sl(2))
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$$[L_n^\pm, L_m^\pm] = (n-m)\, L_{n+m}^\pm + \frac{c^\pm}{12} n^3\, \delta_{n+m}$$
$$[L_n^\pm, W_m^\pm] = (2n-m)\, W_{n+m}^\pm$$
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- IW contraction in large-ℓ limit

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$$[L_n, L_m] = (n-m)L_{n+m} \qquad [L_n, M_m] = (n-m)M_{n+m} + \frac{c}{12}n^3\delta_{n+m}$$

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$$[W_n, W_m] = \text{lgthy}(L, LM, M^2) \qquad [W_n, V_m] = \text{lgthy}(M, M^2) + \frac{c}{12}n^5\,\delta_{n+m}$$

$$\text{HS supertranslations:} \qquad [M_n, M_m] = [M_n, V_m] = [V_n, V_m] = 0$$

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- Same AS algebra obtained directly from isl(3) CS theory

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However, structure in two dimensions different from three:

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Proceed as follows:

- Recap AdS_2 higher spin theories (known)
- Construct flat space spin-2 theory (new)
- Embed flat space spin-2 algebra in higher rank algebra (to do)

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Dilaton gravity in 2d (review: see hep-th/0204253)

Bulk action ($X =$ dilaton):

$$I[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\ \big[XR - U(X)(\nabla X)^2 - 2V(X)\big]$$

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  - linear dilaton vacua: $e^{Q(X)}\,\mathrm{d}X = \mathrm{d}r$ with $Q \propto \int^X U(y)\,\mathrm{d}y$ and

    $$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r - e^{Q(X(r))}\big(w(X(r)) - M\big)\,\mathrm{d}u^2$$

    where $w(X) \propto \int^X e^{Q(y)} V(y)\,\mathrm{d}y$ and $M =$ conserved mass

    generalized Birkhoff theorem: all solutions have Killing vector ∂_u

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Focus for time being on JT with negative Λ (AdS$_2$)

Selected list of models

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten Black Hole (1991)	$-\frac{1}{X}$	$-2b^2X$
4. CGHS (1992)	0	$-2b^2$
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. Black Hole attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A\sinh X$
17. 2D type 0A string Black Hole	$-\frac{1}{X}$	$-2b^2X + \frac{b^2q^2}{8\pi}$
18. exact string Black Hole (2005)	lengthy	lengthy

Gauge theory formulation of Jackiw–Teitelboim model

BF is to JT what CS is to EH

$$I_{\mathrm{BF}}[\mathcal{X},\, A] = \frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}\, F \rangle$$

$F = \mathrm{d}A + A \wedge A$ with $A \in \mathsf{sl}(2, \mathbb{R})$; co-adjoint scalars $\mathcal{X}$

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- EOM $F = 0$ imply torsionlessness and constancy of Ricci-scalar

  schematically:
  $A = e^a P_a + \omega J$ with $[P_a,\, J] = \epsilon_a{}^b P_b$ and $[P_a,\, P_b] = \Lambda\epsilon_{ab}\, J$

$$\text{EOM:}\qquad \underbrace{\mathrm{d}e^a + \epsilon^a{}_b\,\omega\wedge e^b}_{\text{torsionlessness}} = 0 = \underbrace{\mathrm{d}\omega - \tfrac{1}{2}\,\Lambda\,\epsilon_{ab}\,e^a\wedge e^b}_{\text{constancy of Ricci-scalar}}$$

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- choose Euklidean disk with coord's $(\tau,\,\rho)\sim(\tau+\beta,\,\rho)$ and $\rho\in[0,\infty)$
- use convenient parametrization $A=b^{-1}\big(\mathrm{d}+a_\tau\,\mathrm{d}\tau\big)b$, $\mathcal{X}=b^{-1}xb$
- Casimir (mass), $C\sim\langle\mathcal{X}\,\mathcal{X}\rangle\sim\mathrm{Tr}\,(x^2)$, conserved on-shell, $\partial_\tau C=0$

Boundary and integrability conditions for JT

See DG, McNees, Salzer, Valcárcel, Vassilevich '17 and González, DG, Salzer '18

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$$a_\tau = f_\tau\, x + g^{-1}\partial_\tau g$$

with $g = \exp\left(-\tfrac{1}{2}\, y' L_{-1}\right) \exp\left(\ln(y) L_0\right)$ where $f_\tau = 1/y$

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note: boundary action given by

$$I_{\partial\mathcal{M}} \sim \int \mathrm{d}\tau f_\tau\, \mathrm{Tr}\left(x^2\right) \sim \int \mathrm{d}\tau f_\tau\, C$$

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- defining inverse diffeo, $f^{-1}(u) := \tau(u)$ and inserting into Casimir

$$\Gamma|_{F=0}[\tau] = -\frac{k\, \bar{y}}{2\pi} \int_0^\beta \mathrm{d}u\, \left[\dot{\tau}^2 \mathcal{L} + \tfrac{1}{2}\, \{\tau;\, u\}\right] \qquad \{\tau;\, u\} = \frac{\dddot{\tau}}{\dot{\tau}} - \frac{3}{2}\, \frac{\ddot{\tau}^2}{\dot{\tau}^2}$$

yields Schwarzian action, with $k \sim N_{\text{SYK}}$ and $1/\bar{y} \sim J_{\text{SYK}}$

Spin-2 to HS: similar recipe as in 3d

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- after some gymnastics: spin-3 generalization of Schwarzian action (see González, DG, Salzer '18)

Outline

Flat space higher spin gravity in 3d

AdS higher spin gravity in 2d

Flat space spin-2 gravity in 2d

Towards flat space higher spin gravity in 2d

Callan–Giddings–Harvey–Strominger model

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Want interesting flat space spin-2 gravity model in 2d:

- $\Lambda \to 0$ limit of JT boring model (no horizons)

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$$\lim_{\Lambda\to 0} I_{\text{JT}}[X,\, g_{\mu\nu}] = \lim_{\Lambda\to 0} \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[X(R-2\Lambda)\big] = \text{boring}$$

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- or add dilaton-independent term to dilaton potential

$$I_{\widehat{\text{CGHS}}}[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\left[X R - 2\Lambda\right]$$

Note: original CGHS-model/Witten black hole has bulk action

$$I_{\text{CGHS}} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x\sqrt{|g|}\,\Big(X R - \frac{1}{X}\,(\nabla X)^2 - 2\Lambda\, X - \frac{1}{2}\,(\nabla f)^2\Big)$$

eliminate extra scalars ($f=0$) and make Weyl rescaling to get $I_{\widehat{\text{CGHS}}}$

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Reasonable starting point for (Rindler-type) flat space holography

Asymptotic Killing vectors and BMS_2 symmetries

Work in progress with Afshar, González, Salzer, Vassilevich '19

CGHS line-element

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$$\xi = \epsilon(u)\,\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\,\partial_r$$

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isomorphic ($J_n = nM_n$ for $n \neq 0$, $J_0 = M_0$) to warped conformal algebra

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Cangemi–Jackiw gauge theoretic formulation and bc's

- Ansatz for connection and scalar field

$$A = \omega J + e^a P_a + AZ \qquad \mathcal{X} = XZ + X^a \varepsilon_a{}^b P_b + YJ$$

with centrally extended Poincaré$_2$

$$[P_+, \, P_-] = Z \qquad [P_\pm, \, J] = \pm P_\pm$$

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$$a_u = \mathcal{T}P_+ + P_- + \mathcal{P}J \qquad x = \big(\eta' + \mathcal{T}\varepsilon\big)\,P_+ + \varepsilon P_- + \big(\varepsilon' + \mathcal{P}\varepsilon\big)\,J + \eta\,Z$$

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- yields metric in EF-gauge, with same functions $\mathcal{P}, \mathcal{T}$ as before

Consequence of bc's and integrability conditions

Proceed analogously to JT-case:

- bc-preserving gauge trafos

$$\lambda = b^{-1}\big(((\varepsilon^Z)' + \mathcal{T}\varepsilon^-)P_+ + \varepsilon^- P_- + ((\varepsilon^-)' + \mathcal{P}\varepsilon^-)J + \varepsilon^Z Z\big)b$$

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Twisted warped transformation behavior!

Note: in modes ($L_n \leftarrow \mathcal{T}$, $J_n \leftarrow \mathcal{P}$) trafo-behavior above corresponds to twisted warped conformal algebra (see Afshar, Detournay, DG, Oblak '15)

$$[L_n,\, L_m] = (n-m)L_{n+m}$$

$$[L_n,\, J_m] = -mJ_{n+m} + i\,n^2\delta_{n+m}$$

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Twisted warped transformation behavior!

- same transformation behavior follows from asymptotic Killing vectors

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- same transformation behavior follows from asymptotic Killing vectors
- dual field theory (if exists) has twisted warped conformal symmetries

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Proceed analogously to JT-case:

- bc-preserving gauge trafos

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- finite trafos also featured recently in Afshar '19

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Coincides with boundary action derived in 1908.08089

Outline

Flat space higher spin gravity in 3d

AdS higher spin gravity in 2d

Flat space spin-2 gravity in 2d

Towards flat space higher spin gravity in 2d

Outlook on flat space higher spin gravity in 2d

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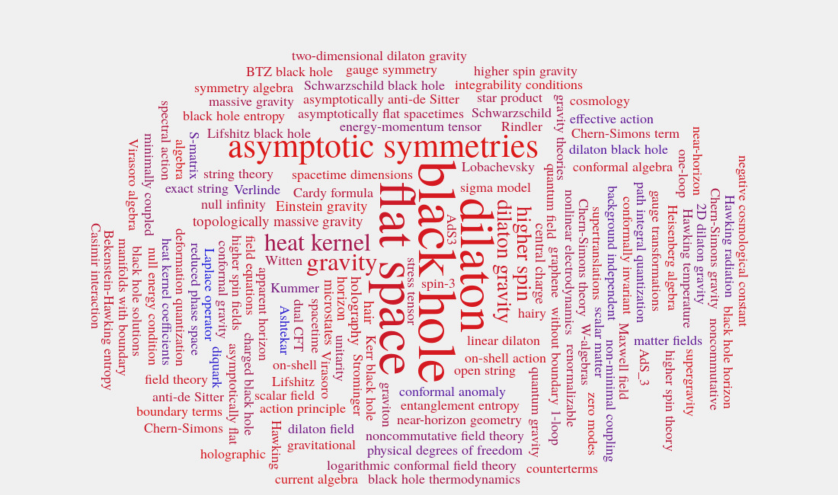
Flat space higher spin gravity in 2d probably exists

Thanks for your attention!

Thanks to

Hamid Afshar, Hernán González, Jakob Salzer and Dima Vassilevich

for collaborations on (flat space) dilaton (higher spin) gravity in 2d!



Bonus-slide

González, DG, Salzer '18

Spin-3 Schwarzian action (zero temperature)

$$I \sim \int \mathrm{d}\tau \left[\frac{f'''}{f'} - \frac{4}{3}\left(\frac{f''}{f'}\right)^2 + \frac{e'''}{e'} - \frac{4}{3}\left(\frac{e''}{e'}\right)^2 - \frac{1}{3}\,\frac{f''e''}{f'e'} \right]$$

with

$$e = s'/f'$$

has $\mathsf{SL}(3)$-invariance

$$s \to \frac{a_{11}s + a_{12}f + a_{13}}{a_{31}s + a_{32}f + a_{33}} \qquad\qquad f \to \frac{a_{21}s + a_{22}f + a_{23}}{a_{31}s + a_{32}f + a_{33}}$$

where a_{ij} are components of $\mathsf{SL}(3)$-matrix

See Marshakov, Morozov '90; ...; Li, Theisen '15