Soft Heisenberg Hair

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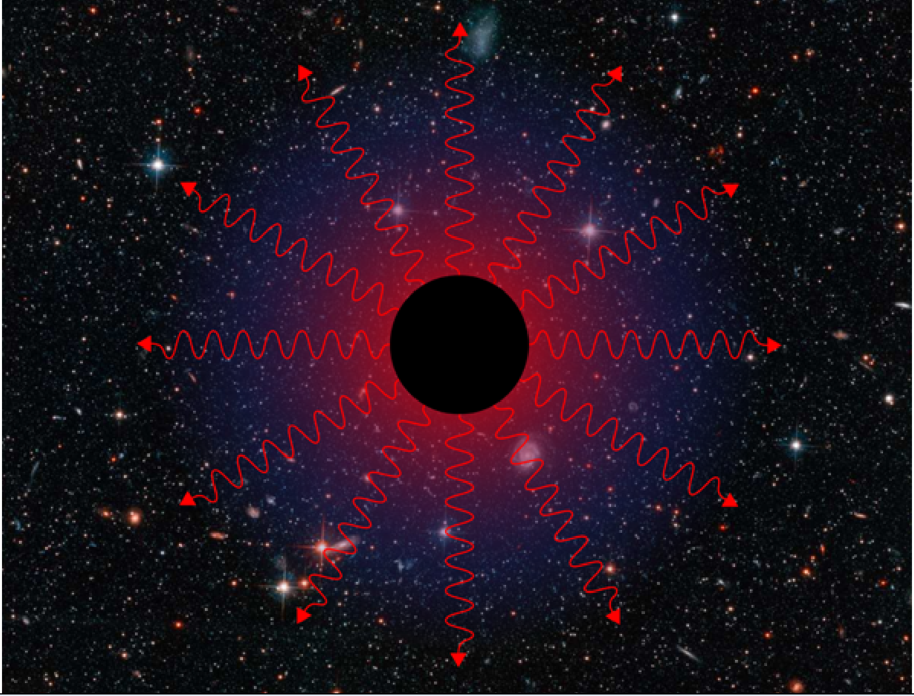
FWF

1603.04824, 1607.00009, 1607.05360, 1611.09783, 1703.02594



Two simple punchlines

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$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
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but also in near horizon physics of (higher spin) gravity theories

2. Black hole microstates identified as specific “soft hair” descendants

based on work with

- Hamid Afshar [IPM Teheran]
- Stephane Detournay [ULB]
- Wout Merbis [TU Wien]
- Blagoje Oblak [ULB / ETH]
- Alfredo Perez [CECS Valdivia]
- Stefan Prohazka [TU Wien]
- Shahin Sheikh-Jabbari [IPM Teheran]
- David Tempo [CECS Valdivia]
- Ricardo Troncoso [CECS Valdivia]

Outline

Motivation

Near horizon boundary conditions

Explicit construction of BTZ microstates

Discussion

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  using CFT_2 symmetries and Cardy formula

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  using CFT_2 symmetries and Cardy formula
- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

  warped CFT: Detournay, Hartman, Hofman '12
  Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- Main idea: consider near horizon symmetries for non-extremal horizons

  Related ideas pursued e.g. by
  - Donnay, Giribet, Gonzalez, Pino '15
  - Hawking, Perry, Strominger '16
  - ...

  Postpone comparison with related approaches after discussing our approach

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- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

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of Rindler metric

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  suggestion in `1511.08687`

  We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

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Example:

$$\Phi(x \to \infty) = 0$$

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Example: Brown-Henneaux type of bc's (aAdS$_3$):

$$\mathrm{d}s^2_{\text{aAdS}} = \mathrm{d}\rho^2 + \big(e^{2\rho}\eta_{\mu\nu} + \gamma_{\mu\nu} + \mathcal{O}(e^{-2\rho})\big)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

with $\delta\gamma =$ arbitrary

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- Local diffeos and gauge trafos fall into three classes:
  1. Trafos that violate bc's (forbidden)
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- Consistency means they are finite, integrable, non-trivial and conserved (in time)

AdS_3 bc's

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- Troessaert (2013): 2 Virasoros plus 2 $u(1)$ current algebras
- Avery–Poojary–Suryanarayana (2013): Virasoro plus $sl(2)$ current algebra
- Donnay–Giribet–Gonzalez–Pino (2015): centerless warped conformal
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Our near horizon bc's simpler than any of the above!

Explicit specification of our bc's in diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d} + \mathfrak{a}_{\pm}(x^0, x^1)\right) b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Standard AdS$_3$ approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\,x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

$sl(2)$: $[L_n,\,L_m] = (n-m)L_{n+m}, \quad n, m = -1, 0, 1$

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- For near horizon purposes diagonal gauge useful:

$$\mathfrak{a} \sim \mathcal{J}(x^0,\, x^1)\, L_0$$

- Precise boundary conditions (ζ: chemical potential):

$$\mathfrak{a} = \left(\mathcal{J}\ \mathrm{d}\varphi + \zeta\ \mathrm{d}v\right)\, L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\ \mathrm{d}\varphi\, L_0$$

and $b = \exp\left(\frac{1}{\zeta}\, L_+\right) \cdot \exp\left(\frac{\rho}{2}\, L_-\right)$. (assume constant ζ for simplicity)

Near horizon metric

Using

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yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f \,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \big[\gamma^2 + \tfrac{2\rho}{a} f(\gamma^2 - \omega^2)\big]\,\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Comments:

- Recover desired near horizon metric

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- $\gamma = \gamma(\varphi)$: "black flower"

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- Zero mode charges: mass and angular momentum

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Background independent result for Chern–Simons yields

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Meaningful near horizon boundary conditions and non-trivial theory!

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

$$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant ζ given by

$$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

$$\partial_v \eta = 0$$

  implying

$$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

$$J_n^\pm = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^\pm(\varphi)$$

What should we expect?

  - Virasoro? (spacetime is locally AdS_3)
  - BMS_3? (Rindler boundary similar to scri)
  - warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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$$\left[J_n^{\pm},\, J_m^{\pm}\right] = \pm\tfrac{1}{2} kn \delta_{n+m,\,0} \qquad \left[J_n^{+},\, J_m^{-}\right] = 0$$

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  Two $\hat{u}(1)$ current algebras with non-zero levels
- Much simpler than CFT_2, warped CFT_2, Galilean CFT_2, etc.
- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\left(J_{-n}^+ + J_{-n}^-\right) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Macroscopic entropy

Using any of the usual methods to determine entropy yields

$$S = 2\pi \left(J_0^+ + J_0^- \right)$$

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e.g. entropy for BTZ black holes in spin-3 gravity:
asymptotic result

$$S = 2\pi\sqrt{2\pi k} \left(\sqrt{\mathcal{L}_+} \cos\left[\frac{1}{3} \arcsin\left(\frac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_+^3}}\mathcal{W}_+ \right) \right] + \sqrt{\mathcal{L}_-} \cos\left[\frac{1}{3} \arcsin\left(\frac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_-^3}}\mathcal{W}_- \right) \right] \right)$$

equivalent to simpler near horizon result above!

Brief list of generalizations

Heisenberg algebras as near horizon symmetries arise not only in AdS_3 Einstein gravity, but also in ...

- ... flat space Einstein gravity in three dimensions
  Afshar, DG, Merbis, Perez, Tempo, Troncoso '16
- ... higher spin gravity in three dimensions
  DG, Perez, Prohazka, Tempo, Troncoso '16
- ... higher derivative gravity in three dimensions
  Setare, Adami '16
- ... general relativity (in four dimensions)
  Afshar, DG, Sheikh-Jabbari '16
- ... flat space higher spin gravity in three dimensions
  Ammon, Grumiller, Prohazka, Riegler, Wutte '17

Conclusions about near horizon symmetry algebra fairly general!

Outline

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- Call this "near horizon symmetry algebra" (note: independent from ℓ)

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- Will exploit this property to provide cut-off on soft hair spectrum!

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- Microstates = all states in near horizon Hilbert space obeying equations above

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We are now ready to identify all BTZ microstates

- Vector space $\mathcal{V}_{\mathcal{B}}$ of BTZ microstates defined by

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- Agrees with Bekenstein–Hawking and Cardy formula

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Near horizon boundary conditions

Explicit construction of BTZ microstates

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- Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16: introduced near horizon bc's we use; did not attempt construction of microstates (but does Cardy-type of counting)

Generalization to four dimensions

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Compare with near horizon construction of Donnay, Giribet, Gonzalez, Pino '15

- Near horizon algebra similar to but different from BT-BMS$_4$:

$$[\mathcal{Y}_n^\pm,\, \mathcal{Y}_m^\pm] = (n-m)\,\mathcal{Y}_{n+m}^\pm$$
$$[\mathcal{Y}_l^+,\, \mathcal{T}_{(n,m)}] = -n\,\mathcal{T}_{(n+l,\,m)}$$
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recovers 4d algebra above by "Sugawara construction"

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- Making AKVs in DGGP state-dependent to leading order relates their canonical boundary charges to Heisenberg boundary charges

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$$[\mathcal{Y}_l^-,\,\mathcal{T}_{(n,m)}] = -m\,\mathcal{T}_{(n,\,m+l)}$$

- Intriguing algebraic observation: introducing again

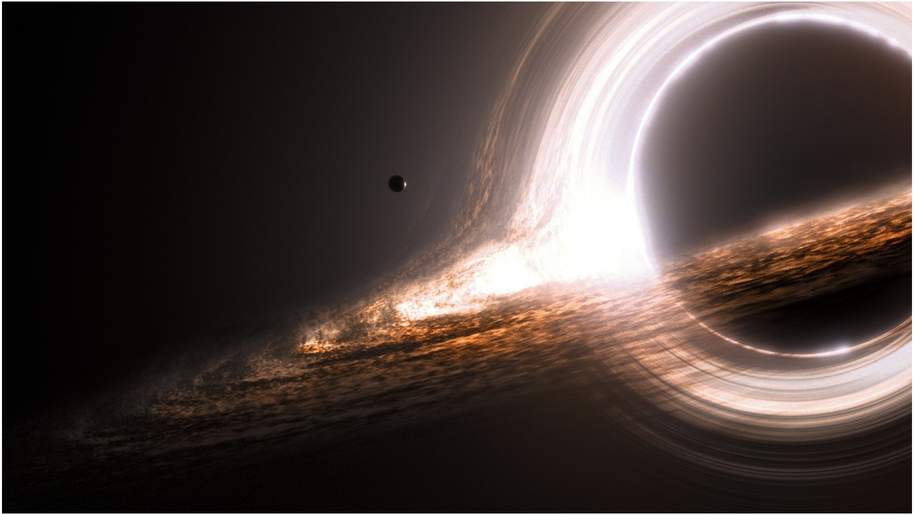
$$[\mathcal{J}_n^\pm,\,\mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n,-m} = -[\mathcal{K}_n^\pm,\,\mathcal{K}_m^\pm]$$

  recovers 4d algebra above by "Sugawara construction"

$$\mathcal{T}_{(n,\,m)} = \big(\mathcal{J}_n^+ + \mathcal{K}_n^+\big)\big(\mathcal{J}_m^- + \mathcal{K}_m^-\big)$$
$$\mathcal{Y}_n^\pm = \sum_{p\in\mathbb{Z}} \big(\mathcal{J}_{n-p}^\pm + \mathcal{K}_{n-p}^\pm\big)\big(\mathcal{J}_p^\pm - \mathcal{K}_p^\pm\big)$$

- Making AKVs in DGGP state-dependent to leading order relates their canonical boundary charges to Heisenberg boundary charges
- Indicates existence of soft Heisenberg hair in 4d

Microstates of non-extremal Kerr?



Main challenge: how to provide (controlled) cut-off on soft hair spectrum in four dimensions?

Thanks for your attention!

- H. Afshar, D. Grumiller and M.M. Sheikh-Jabbari "Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes," 1607.00009.
- H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso "Soft Heisenberg hair on black holes in three dimensions," Phys.Rev. **D93** (2016) 101503(R); 1603.04824.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

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Near horizon boundary conditions natural for near horizon observer

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Punchline: our proposal is Bohr-type quantization of spectrum

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- Spectral flow and discrete conic spaces generated by $\mathcal{J}_r^\pm$ ($r = 1, 2, \dots c-1$), the "horizon fluff"

On log corrections

- Semi-classically

$$S = S_0 + \# \cdot \ln S_0 + \mathcal{O}(1)$$

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- Mismatch in coefficients; not sure yet if bug or feature