

Flat Space Holography

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Quantum vacuum and gravitation
Mainz, June 2015



based on work w. Afshar, Bagchi, Basu, Detournay, Fareghbal,
Gary, Riegler, Rosseel, Salzer, Sarkar, Schöller, Simon, ...

Some of our papers on flat space holography

-  A. Bagchi, R. Basu, D. Grumiller and M. Riegler,
“Entanglement entropy in Galilean conformal field theories and flat holography,”
Phys. Rev. Lett. **114** (2015) 11, 111602 [arXiv:1410.4089].
-  H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel,
“Spin-3 Gravity in Three-Dimensional Flat Space,”
Phys. Rev. Lett. **111** (2013) 12, 121603 [arXiv:1307.4768].
-  A. Bagchi, S. Detournay, D. Grumiller and J. Simon,
“Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space,”
Phys. Rev. Lett. **111** (2013) 18, 181301 [arXiv:1305.2919].
-  A. Bagchi, S. Detournay and D. Grumiller,
“Flat-Space Chiral Gravity,”
Phys. Rev. Lett. **109** (2012) 151301 [arXiv:1208.1658].
-  S. Detournay, D. Grumiller, F. Schöller and J. Simon,
“Variational principle and 1-point functions in 3-dimensional flat space Einstein gravity,” Phys. Rev. D **89** (2014) 8, 084061 [arXiv:1402.3687].

Outline

Motivations

Holography basics

Flat space holography

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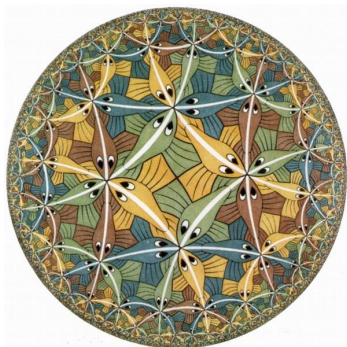
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The aim of the program is to analyze from a modern perspective QFT methods, like renormalization group and conformal anomalies, and their applications in astrophysics and cosmology with special focus on black hole physics and the study of analogue systems.

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Main question: how general is holography?

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- ▶ To what extent do (previous) lessons rely on the particular constructions used to date?
- ▶ Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012

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- ▶ originally holography motivated by unitarity
- ▶ plausible AdS/CFT-like correspondence could work non-unitarily
- ▶ AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson '08; Skenderis, Taylor, van Rees '09; Henneaux, Martinez, Troncoso '09; Maloney, Song, Strominger '09; DG, Sachs/Hohm '09; Gaberdiel, DG, Vassilevich '10; ... DG, Riedler, Rosseel, Zojer '13
- ▶ recent proposal by Vafa '14

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- ▶ Can we establish a flat space holographic dictionary?

the answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., '12-'15

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- ▶ Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work at least in 2+1 dimensions

Gary, DG Rashkov '12; Afshar et al '12; Gutperle et al '14-'15; Gary, DG, Prohazka, Rey '14; ...

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- ▶ Address questions above in simple class of 3D toy models
- ▶ Exploit gauge theoretic Chern–Simons formulation
- ▶ Restrict to kinematic questions, like (asymptotic) symmetries

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Address these issues in 3D!



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- ▶ Define quantum gravity by its dual field theory

Interesting dichotomy:

- ▶ Either dual field theory exists \rightarrow useful toy model for quantum gravity
- ▶ Or gravitational theory needs UV completion (within string theory) \rightarrow indication of inevitability of string theory

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This talk:

- ▶ Remain agnostic about dichotomy
- ▶ Focus on generic features of dual field theories that do not require string theory embedding

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AdS₃ gravity

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Interesting generic constraints from CFT₂!

e.g. [Hellerman '09](#), [Hartman, Keller, Stoica '14](#)

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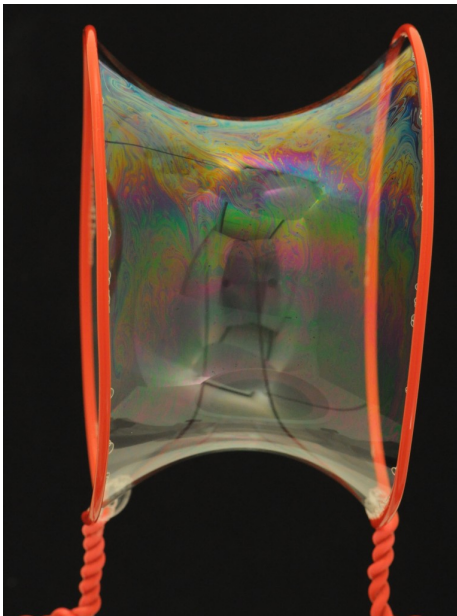
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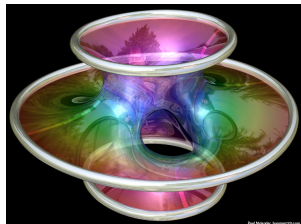
Caveat: while there are many string compactifications with AdS₃ factor, applying holography just to AdS₃ factor does not capture everything!

Picturesque analogy: soap films



Both soap films and Chern–Simons theories have

- ▶ essentially no bulk dynamics
- ▶ highly non-trivial boundary dynamics
- ▶ most of the physics determined by boundary conditions
- ▶ esthetic appeal (at least for me)



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Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true \Rightarrow must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

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- ▶ Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \quad M_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$$

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- ▶ Make Inönü–Wigner contraction $\ell \rightarrow \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m, 0}$$

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- ▶ Example where it does not work: highest weight conditions

Flat space Einstein gravity as $isl(2)$ Chern–Simons theory

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- ▶ AdS gravity in CS formulation: $sl(2) \oplus sl(2)$ gauge algebra

Flat space Einstein gravity as $\mathfrak{isl}(2)$ Chern–Simons theory

- ▶ AdS gravity in CS formulation: $\mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$ gauge algebra
- ▶ Flat space: $\mathfrak{isl}(2)$ gauge algebra

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \langle \mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \rangle$$

with $\mathfrak{isl}(2)$ connection ($a = 0, \pm 1$)

$$\mathcal{A} = e^a M_a + \omega^a L_a$$

$\mathfrak{isl}(2)$ algebra (global part of BMS/GCA algebra)

$$[L_a, L_b] = (a - b)L_{a+b}$$

$$[L_a, M_b] = (a - b)M_{a+b}$$

$$[M_a, M_b] = 0$$

Note: e^a dreibein, ω^a (dualized) spin-connection

Bulk EOM: gauge flatness

$$\mathcal{F} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = 0$$

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- ▶ Flat space boundary conditions: $b(r) = \exp(\frac{1}{2} r M_{-1})$ and

$$a(u, \varphi) = (M_1 - M(\varphi)M_{-1}) du + (L_1 - M(\varphi)L_{-1} - N(u, \varphi)M_{-1}) d\varphi$$

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- ▶ metric

$$g_{\mu\nu} \sim \frac{1}{2} \text{tr} \langle \mathcal{A}_\mu \mathcal{A}_\nu \rangle \quad \rightarrow \quad ds^2 = M du^2 - 2 du dr + 2N du d\varphi + r^2 d\varphi^2$$

Selected recent results

- ▶ Classical saddle points of flat space Einstein gravity:

$$ds^2 = M du^2 - 2 du dr + 2N du d\varphi + r^2 d\varphi^2$$

Minkowski: $M = -1$, $N = 0$

Flat-space cosmologies (FSC): $M > 0$, $N \neq 0$ (analogue of BTZ)

Note: different way to write FSC:

$$ds^2 = -d\tau^2 + \frac{(E\tau)^2 dx^2}{1 + (E\tau)^2} + (1 + (E\tau)^2) \left(dy + \frac{(E\tau)^2}{1 + (E\tau)^2} dx \right)^2$$

Time-dependent background (Cornalba, Costa '02)

Penrose diagram: like Schwarzschild turned by 90 degrees

Obtained as limit from region between inner/outer BTZ horizons
(Barnich, Gomberoff, Gonzalez '12)

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Galilean conformal field theory! (Bagchi et al, Barnich et al)

Algebra:

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M_n : “supertranslations”, L_n : “superrotations”

Gravity interpretation: ultra-relativistic boost (AdS bdy maps to scri)

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Works! (Bagchi, Detournay, Fareghbal, Simon '13, Barnich '13)

$$S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2h_M}} = S_{\text{GCFT}}$$

Also works as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

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$$ds^2 = M du^2 - 2 du dr + 2N du d\varphi + r^2 d\varphi^2$$

- ▶ Dual field theory?
- ▶ Microstate counting?
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Exists! (Bagchi, Detournay, DG, Simon '13)

Small temperatures: flat space thermodynamically stable

Large temperatures: FSC thermodynamically stable

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Works! (Detournay, DG, Schöller, Simon '14)

Interesting technical detail: boundary term half of GHY

$$\Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{|g|} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{|\gamma|} K$$

$\delta\Gamma = 0$ for all metric variations that preserve flat space bc's

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Works! (Bagchi, Basu, DG, Riegler '14)

$$S_{\text{EE}}^{\text{GCFT}} = \underbrace{\frac{c_L}{6} \ln \frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6} \frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

Calculation on gravity side confirms result above
(using Wilson lines in CS formulation)

Generalizations & open issues

Recent generalizations:

- ▶ adding chemical potentials

Works! (Gary, DG, Riegler, Rosseel '14)

In CS formulation:

$$A_0 \rightarrow A_0 + \mu$$

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Conformal CS gravity at level $k = 1$ with flat space boundary conditions conjectured to be dual to chiral half of monster CFT.

Action (gravity side):

$$I_{\text{CSG}} = \frac{k}{4\pi} \int d^3x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} (\partial_\mu \Gamma^\sigma_{\nu\rho} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho})$$

Partition function (field theory side, see Witten '07):

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

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- ▶ generalization to supergravity

Works! (Barnich, Donnay, Matulich, Troncoso '14)

Asymptotic symmetry algebra = super-BMS₃

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Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13)

New type of algebra: W-like BMS (“BMW”)

$$[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M}(n - m)\Lambda_{n+m} \\ - \frac{96(c_L + \frac{44}{5})}{c_M^2}(n - m)\Theta_{n+m} + \frac{c_L}{12}n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}$$

$$[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M}(n - m)\Theta_{n+m} \\ + \frac{c_M}{12}n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}$$

$$[L, L], [L, M], [M, M] \text{ as in BMS}_3 \quad [L, U], [L, V], [M, U], [M, V] \text{ as in isl}(3)$$

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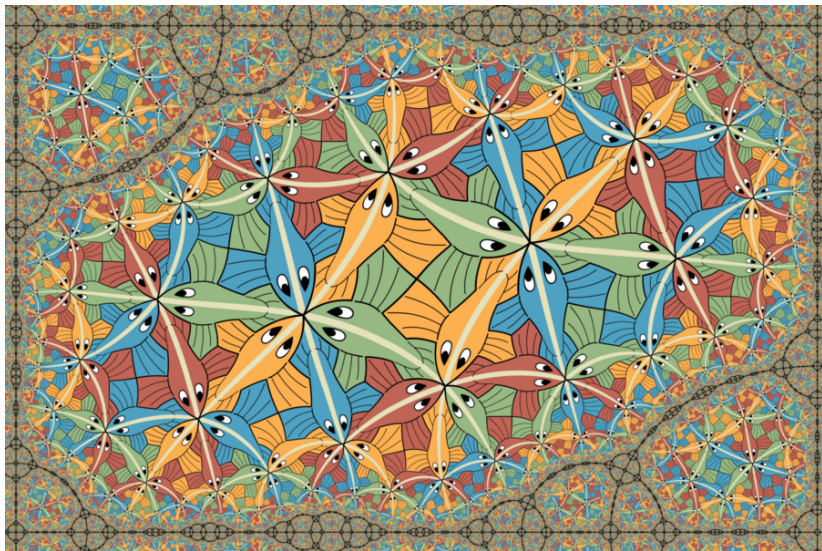
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- ▶ holography more general than AdS/CFT
- ▶ (when) does it work even more generally?

Thanks for your attention!



Vladimir Bulatov, M.C. Escher Circle Limit III in a rectangle