Soft Heisenberg Hair

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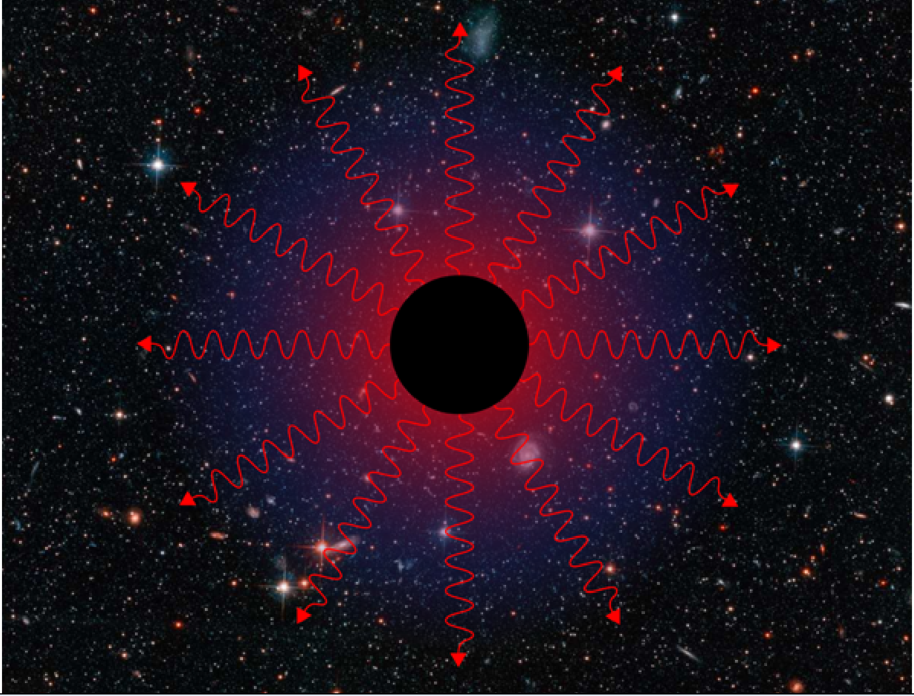

Two simple punchlines

1. Heisenberg algebra

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fundamental not only in quantum mechanics
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but also in near horizon physics of gravity theories

2. Black hole microstates identified as specific “soft hair” descendants

based on work with

- Hamid Afshar, Shahin Sheikh-Jabbari [IPM Teheran]
- Martin Ammon [U. Jena]
- Stephane Detournay, Wout Merbis, Stefan Prohazka, Max Riegler [ULB]
- Hernán González, Philip Hacker, Raphaela Wutte [TU Wien]
- Alfredo Perez, David Tempo, Ricardo Troncoso [CECS Valdivia]
- Hossein Yavartanoo [ITP Beijing]

Outline

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Motivation

Problems (and possible resolutions)

Near horizon boundary conditions and soft hair

Proposal for semi-classical BTZ microstates

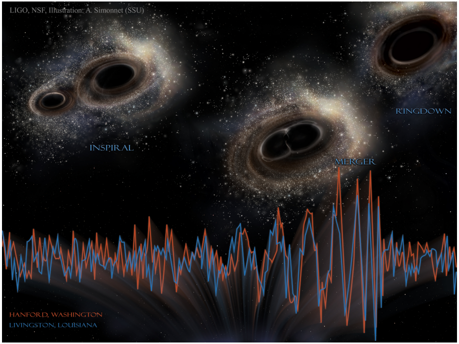
New entropy formula for Kerr black holes

Outlook

Overall motivation: Quantum Gravity

Classical general relativity (GR) well-tested directly

- tests of equivalence principle (redshifts, GPS, etc.)
- perihelion shifts, light-bending, Shapiro time-delay (classical tests)
- gravitational lensing
- frame-dragging, Lense–Thirring (need to be more accurate)
- binary pulsars (indirect detection of gravity waves); $\alpha_3 \leq 4 \cdot 10^{-20}$
- black hole/neutron star mergers (direct detection of gravity waves)



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Quantum GR almost untested

- speed of gravity waves close to speed of light $|\Delta c|/c \leq 10^{-16}$
- no modified dispersion relations up to Planck scale (FERMI satellite)
- low energy theory: quantum field theory (QFT)+(semi-)classical GR
- direct quantum gravity tests hard since Planck energy too high, $E_{\text{Planck}}/E_{\text{LHC}} \sim 10^{16}$

Know little about quantum gravity, but both from theory and experimental side it should reduce to GR + QFT semi-classically

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Semi-classical GR well-tested indirectly

- from above: GR correct classical theory
- from particle physics: QFT works
  example: $g_{\text{ex}}/2 = 1.00115965218(073)$, $g_{\text{th}}/2 = 1.00115965218(178)$
- synthesis: plausible that semi-classical GR should work

> Semi-classical predictions like Hawking effect most likely true!
> Consequence: black holes have entropy!

Motivation 1: universality of black hole entropy

Bekenstein–Hawking

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$$S = S_{\text{BH}} - q\,\ln S_{\text{BH}} + \mathcal{O}(1) \qquad q = \text{number depending on matter}$$

currently "template for experimental results" in quantum gravity

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[at least any theory of quantum gravity claiming to reproduce (semi-)classical Einstein gravity in limit of small Newton constant]

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Perhaps no need for full knowledge of quantum gravity to account microscopically for black hole entropy (of sufficiently large black holes)

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$$S_{\text{Cardy}} = 2\pi\left(\sqrt{c\Delta^+/6} + \sqrt{c\Delta^-/6}\right) = \frac{A}{4G} = S_{\text{BH}}$$

  c: left/right central charges of CFT_2
  $\Delta^\pm$: left/right energies of state whose entropy is counted

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

  warped CFT: Detournay, Hartman, Hofman '12
  Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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Kerr/CFT: Guica, Hartman, Song, Strominger '09; Compere '12

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Hope: near horizon symmetries allow for Cardyology

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additional motivation: information loss

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Perhaps no need for full knowledge of quantum gravity to construct microstates (of sufficiently large non-extremal black holes)
[at least for some observer, not necessarily an asymptotic one]

Synthesis of the three motivations: soft hair

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- General relativity with (asymptotic) boundaries:
  (locally) diffeomorphic geometries may be physically inequivalent

  Famous example: BTZ black hole is locally AdS_3, but canonical boundary charges (e.g. mass, angular momentum) differ
  Bañados, Henneaux, Teitelboim, Zanelli '93

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Donnay, Giribet, Gonzalez, Pino '16
Afshar, Detournay, Grumiller, Merbis, Perez, Tempo, Troncoso '16

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Hope: soft hair could address black hole entropy puzzles and microstates in a semi-classical framework

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Problem: how?

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Resolving the ‘how’-questions easier in simpler models

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Consider as toy model Einstein gravity in three dimensions with negative cosmological constant

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Same conceptual problems as in higher dimension, but technically more manageable

Outline

Properties of Einstein gravity in 2+1 dimensions with negative cc (AdS$_3$)

- Second order bulk action:

$$I_{\text{EH}} = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{-g}\,\Big(R + \frac{2}{\ell^2}\Big)$$

G: Newton constant in 2+1 dimensions; ℓ: AdS radius

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- Spectrum of physical states includes BTZ black holes

$$\mathrm{d}s^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\,\Big(\mathrm{d}\varphi - \frac{r_+r_-}{\ell r^2}\,\mathrm{d}t\Big)^2$$

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- BTZ BH entropy given by Bekenstein–Hawking

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- BTZ BH entropy given by Bekenstein–Hawking and Cardy formula

$$S_{\text{BH}} = \frac{A}{4G} = \frac{2\pi r_+}{4G} = 2\pi\,\Big(\sqrt{c\Delta^+/6}+\sqrt{c\Delta^-/6}\Big)$$

  $\Delta^\pm = (r_+\pm r_-)^2/(16\ell G) \propto \ell M \pm J$ (M: mass, J: angular momentum)

Near horizon boundary conditions

See Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16 for details

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- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

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$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

  Meaning of coordinates:
  - ρ: radial direction ($\rho = 0$ is horizon)
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- This is somewhat unusual, but convenient for our purposes!

Explicit form of our boundary conditions in metric formulation

Note: everything much simpler in Chern–Simons formulation!

Boundary conditions as near horizon expansion of metric

$$g_{tt} = -a^2 r^2 + \mathcal{O}\left(r^3\right)$$

$$g_{\varphi\varphi} = \gamma^2 + \left(\gamma^2 - \ell^2\omega^2\right)\frac{r^2}{\ell^2} + \mathcal{O}\left(r^3\right)$$

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$$g_{rr} = 1 + \mathcal{O}\left(r^2\right) \quad g_{rt} = \mathcal{O}\left(r^2\right) \quad g_{r\varphi} = \mathcal{O}\left(r^2\right)$$

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$$g_{t\varphi} = \tfrac{1}{4}\,a\omega r^2 - \tfrac{1}{2}\,a\omega\ell^2 + \mathcal{O}\left(\tfrac{1}{r}\right)$$

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Boundary conditions in Chern–Simons formulation

$$A^\pm = b_\pm^{-1}\big(\,\mathrm{d} + \mathfrak{a}^\pm\big) b_\pm$$

with fixed $\mathfrak{sl}_2$ group element

$$b_\pm = \exp\Big(\pm\frac{r}{2\ell}\left(L_1 - L_{-1}\right)\Big)$$

and 1-form ($\mathcal{J}^\pm = \gamma/\ell \pm \omega$)

$$\mathfrak{a}^\pm = L_0\left(\pm\mathcal{J}^\pm\,\mathrm{d}\varphi - a\,\mathrm{d}t\right) \qquad \delta\mathcal{J}^\pm \neq 0 \quad \delta a = 0$$

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Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \frac{i}{kn}\,(J_{-n}^+ + J_{-n}^-)\text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

yields Heisenberg algebra (with Casimirs $X_0,\, P_0$)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Map explains word "Heisenberg" in title and provides first punchline

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  Whole spectrum (subject to reality) compatible with regularity!

  Could be used as defining property of our bc's

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- Near horizon Hamiltonian $H \sim J_0^+ + J_0^-$ commutes with all $J_n^\pm$

  Near horizon Hamiltonian defined as diffeo charge generated by unit translations ∂_v in (advanced) time direction

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- Consequence: soft hair!

$$H|\psi\rangle = E|\psi\rangle \quad \Rightarrow \quad H|\tilde{\psi}\rangle = E|\tilde{\psi}\rangle$$

where state $\tilde{\psi}$ is state ψ dressed arbitrarily with soft hair

$$|\tilde{\psi}\rangle = \prod_{n_i^{\pm} \in \mathbb{Z}^+} J^+_{n_i^+} J^-_{n_i^-} |\psi\rangle$$

Explains word "soft hair" in title

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- Entropy formula remarkably simple

$$S = 2\pi \left(J_0^+ + J_0^-\right) = T^{-1}\, H$$

also remarkably universal:
generalizes to flat space, higher spins, higher derivatives!

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- Relations to asymptotic Virasoro charges $L^\pm$ and sources $\mu^\pm$

$$L \sim J^2 + J' \qquad \mu' - \mu J \sim a$$

Twisted Sugawra construction emerges! (yields Brown–Henneaux c)

Outline

Assumptions

For technical details see Afshar, DG, Sheikh-Jabbari, Yavartanoo '17

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   Needed due to relations like

$$\mathcal{J}_{cn} \sim \mathcal{W}_n^0$$

   Note non-local relation

$$\mathcal{W} \sim e^{-2\int \mathcal{J}}$$

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Note twisted periodicity conditions

$$\mathcal{W}^\nu(\varphi + 2\pi) = e^{-2\pi\nu i}\, \mathcal{W}^\nu(\varphi)$$

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   Justifiable through explicit stringy construction in D1-D5 system

   Maldacena, Maoz '00; Lunin, Maldacena, Maoz '02

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   Justification 2: gives nice result

List of all semi-classical BTZ black hole microstates

- Given a BTZ black hole with mass M and angular momentum J (as measured by asymptotic observer) define parameters

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BTZ black hole entropy from counting all semi-classical microstates

We proposed (after some Bohr-type semi-classical quantization conditions) explicit set of BTZ black hole microstates

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$$S = k \log W = \ln N = \ln p(c\Delta^{+}) + \ln p(c\Delta^{-})$$

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- Solved long ago by Hardy, Ramanujan; asymptotic formula (large N):

$$\ln p(N) = 2\pi\sqrt{N/6} - \ln N + \mathcal{O}(1)$$

Dyson '87 *The seeds from Ramanujans garden have been blowing on the wind and have been sprouting all over the landscape.*

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- Leading order coincides with Bekenstein–Hawking/Cardy formula!

BTZ black hole entropy from counting all semi-classical microstates

We proposed (after some Bohr-type semi-classical quantization conditions) explicit set of BTZ black hole microstates

Now let us count these microstates (not "Cardyology" but "Hardyology")

- Straightforward combinatorial problem: partition of integers $p(c\Delta^{\pm})$
- Entropy given by Boltzmann's formula

$$S = k\,\log W = \ln N = \ln p(c\Delta^{+}) + \ln p(c\Delta^{-})$$

- Solved long ago by Hardy, Ramanujan; asymptotic formula (large N):

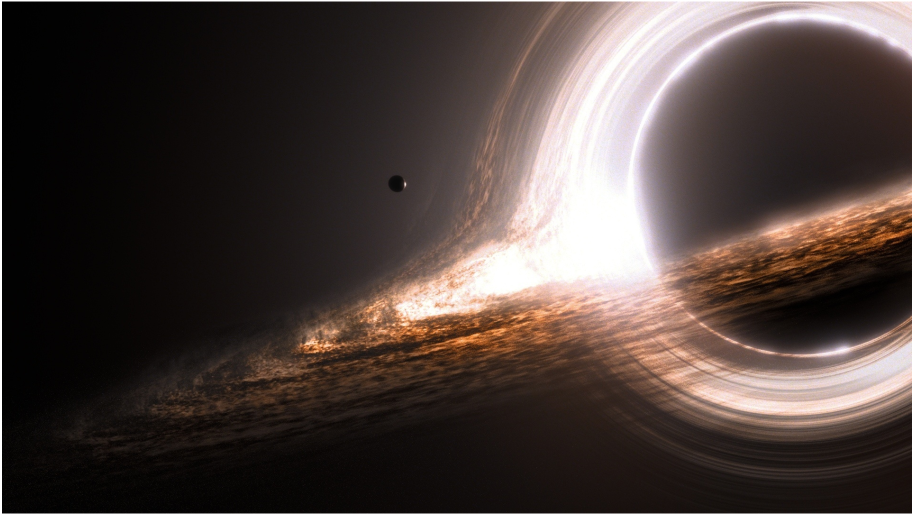
$$\ln p(N) = 2\pi\sqrt{N/6} - \ln N + \mathcal{O}(1)$$

- Our final result for semi-classical BTZ black hole entropy is

$$S = 2\pi\Big(\sqrt{c\Delta^{+}/6} + \sqrt{c\Delta^{-}/6}\Big) - \ln(c\Delta^{+}) - \ln(c\Delta^{-}) + \mathcal{O}(1)$$

- Leading order coincides with Bekenstein–Hawking/Cardy formula!
- Subleading log corrections also turn out to be correct!

Outline



Gargantua, the most popular black hole (simulations for Interstellar, see 1502.03808)

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In particular, does it work in four spacetime dimensions, for astrophysical black holes?

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Check the statements above!
If true: non-trivial hint that story may work!

Near horizon symmetries of Kerr black holes

Are near horizon symmetries representable in terms of $\mathfrak{u}(1)$ current algebras?

Donnay et al '15 found for Kerr near horizon symmetry algebra:

$$[L_n^\pm,\, L_m^\pm] = (n-m)\, L_{n+m}^\pm$$

$$[L_n^+,\, T_{(m,\,k)}] = -m\, T_{(n+m,\,k)} \quad [L_n^-,\, T_{(m,\,k)}] = -k\, T_{(m,\,n+k)}$$

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Sugawara-deconstruction into four $\mathfrak{u}(1)$ current algebras possible:

$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = -[\mathcal{K}_n^\pm,\, \mathcal{K}_m^\pm] = \tfrac{n}{2}\, \delta_{n+m,\,0}$$

with (see Afshar et al '16)

$$T_{(n,\,m)} = \big(\mathcal{J}_n^+ + \mathcal{K}_n^+\big)\big(\mathcal{J}_m^- + \mathcal{K}_m^-\big) \qquad L_n^\pm = \sum_p \big(\mathcal{J}_{n-p}^\pm + \mathcal{K}_{n-p}^\pm\big)\big(\mathcal{J}_p^\pm - \mathcal{K}_p^\pm\big)\,.$$

More convenient basis (see González et al '17)

$$J_n^\pm := \mathcal{J}_n^\pm + \mathcal{K}_n^\pm \qquad K_n^\pm := \mathcal{J}_n^\pm - \mathcal{K}_n^\pm$$

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Is entropy expressible in simple way in terms of 0-modes of these algebras?

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- Require

$$J_0^+ + J_0^- + K_0^+ + K_0^- = 2M$$

  - Chirally symmetric combination, so should not depend on angular momentum
  - Dimensional analysis: must be linear in mass
  - Schwarzschild limit: $J_0^+ = J_0^-$ required
  - Fixes uniquely equation above!

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- Get four algebraic equations in four variables, with unique solution (up to relabellings):

$$J_0^\pm = \frac{1}{2}\big(M + \sqrt{M^2 - a^2} \pm ia\big) \qquad K_0^\pm = \frac{1}{2}\big(M - \sqrt{M^2 - a^2} \pm ia\big)$$

Notational alert: $a = J/M$ is here the Kerr parameter, NOT Rindler acceleration!

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zero mode combination	black hole quantity	physical interpretation
$J_0^+ + J_0^-$	r_+	black hole radius
$K_0^+ + K_0^-$	r_-	Cauchy horizon radius
$J_0^\pm + K_0^\pm$	M	black hole mass
$J_0^+ - K_0^+$	$\sqrt{M^2 - a^2 + n^2}$	extremality parameter
$J_0^+ - J_0^-$	$ia = iJ/M$	Kerr parameter
$\boldsymbol{J_0^+ J_0^-}$	$\boldsymbol{S/(4\pi)}$	**black hole entropy**
$K_0^+ K_0^-$	$S_{\text{inner}}/(4\pi)$	'inner horizon entropy'
$J_0^+ K_0^+ - J_0^- K_0^-$	iJ	angular momentum
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Entropy formula works for all cases studied so far!

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  Perhaps relation to tensionless strings where same algebra arises?
  Bagchi et al '13, '15, '16, '17

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- Soft resolution of information loss problem?
- More generic black holes than BTZ or Kerr?

Thanks for your attention!