Daniel Grumiller March 24^{th} 2021

Gravity and holography in lower dimensions II

(4.1) **Soft hair and information loss**
Assume it is correct that soft hair excitations account for the entropy of black holes. What are the potential difficulties with such a proposal and how could they be overcome?

(4.2) **Transformation of boundary fields**
Show that the Schwarzian transformation behavior

$$\delta_\xi \mathcal{L} = \xi \mathcal{L}' + 2\xi' \mathcal{L} - \frac{1}{2}\,\xi'''$$

is compatible with the twisted Sugawara construction $\mathcal{L} = \frac{1}{4}\,(\Phi')^2 + \frac{1}{2}\,\Phi''$ provided Φ transforms analogously to entanglement entropy in a CFT_2, i.e., like an anomalous scalar field. Moreover, show that $X' = e^{-\Phi}$ transforms like a non-anomalous scalar field and $Y = -\frac{1}{2}\,\Phi'$ like an anomalous vector field under ξ.

(4.3) **Naive ultrarelativistic limit of bosonic strings**
Take the bosonic string spectrum

$$X^\mu_\pm(t \pm \sigma) = \frac{x^\mu}{2} + \frac{\ell_s^2}{2}\,p^\mu_\pm(t \pm \sigma) + \frac{\ell_s}{\sqrt{2}} \sum_{n\neq 0} \frac{\alpha^\pm_{-n}}{in}\, e^{in(t\pm\sigma)}$$

and perform the naive ultrarelativistic limit $t \to \varepsilon t$, $\sigma \to \sigma$ and $\varepsilon \to 0$. Compare the result with the mode expansion of the near horizon boundary field Φ discussed in the lectures.

These exercises are due on April 27^{th} 2021.

Hints/comments:

- Recall what "soft" means and consider a black hole of fixed mass and angular momentum when discussing this issue.

- Remember that entanglement entropy in a CFT_2 transforms as

$$\delta_\xi S = \xi S' - \#\, \xi'$$

  with some number # that depends on the central charge. This is what is meant by "anomalous scalar field" transformation behavior. Determine this number for Φ so that everything works out. If the number vanishes, $\# = 0$, we have instead a non-anomalous scalar field. This should happen for X (possibly up to an irrelevant sign). Finally, an anomalous vector field transforms as

$$\delta_\xi V = \xi V' + \xi' V + \#\, \xi''$$

  where # is another number. This should happen for Y. Note that the fields in this exercise, Φ, X, Y, are the boundary fields that appear in a Gauss decomposition of the Brown–Henneaux connection.

- This exercise is too short to deserve any hints. You find the mode expansion of Φ either in my lecture video or in the paper with Wout, `1906.10694`.