Daniel Grumiller May 25$^{\text{th}}$ 2020

Black Holes II — Exercise sheet 8

(18.1) **Hawking temperature and black hole evaporation**
Write the Hawking temperature $T = \kappa/(2\pi)$ for a stationary black hole with surface gravity $\kappa > 0$ in SI units, i.e., reintroducing Newton's constant G, Planck's constant $\hbar$, Boltzmann's constant k_B and the vacuum speed of light c. Which of these constants appear in the Hawking temperature? What does this tell you about the physics of black hole evaporation? Perform (in units of your choice) an estimate how long it takes for a solar mass black hole to evaporate completely (plus/minus an order of magnitude is sufficient accuracy). Compare the result to the current age of the Universe. How light does a black hole have to be if it was formed shortly after the Big Bang and evaporates right now?

(18.2) **Unruh temperature and acceleration**
How strong does a detector have to accelerate (in meters/second^2) to measure Unruh radiation at a temperature of 1 Kelvin? Let us worry in the rest of this exercise about consistency of Unruh temperature and address the following questions. Do all observers agree that an accelerated detector clicks (i.e., detects Rindler quanta at the Unruh temperature), in particular a Minkowski observer? Does Unruh radiation not violate energy conservation? The equivalence principle equates acceleration and uniform gravitational fields (such as the one on Earth); does this mean observers near Earth should observe Unruh radiation? What happens if a detector does not accelerate forever (and thus sees a Rindler horizon), but only for some finite time?

(18.3) **Boulware vacuum and absence of Hawking radiation**
The Boulware vacuum is defined to be a state without any radiation at $\mathscr{I}^\pm$. Discuss the behavior of the Boulware state near a black hole horizon, in particular regularity of the stress-energy tensor in global coordinates (such as Kruskal).

These exercises are due on June 9$^{\text{th}}$ 2020.

Hints:

- For the first part just remember the SI unit of temperature and reintroduce all the conversion factors (e.g. you should know on general grounds which powers of k_B and G must appear and can then deduce the powers of $\hbar$ and c). As for the interpretation, just remind yourself what it means for a physical law for either of these conversion factors to appear — e.g. the appearance of k_B means you are dealing with thermodynamics, converting some energy into temperature units.
  For the final part estimate the mass loss per time, $\mathrm{d}M/\,\mathrm{d}t$, by equating it to the radiated power P, which in turn is given by $P \propto AT^4$, where A is the surface area of the radiating object (i.e., of the black hole) and the T^4-factor should be familiar from the Stefan–Boltzmann law in four spacetime dimensions.

- Again the first part consists of converting a simple result, $T = a/(2\pi)$, into SI units. The qualitative questions can be addressed without doing actual calculations, but some of them may seem quite non-trivial. As always, feel free to use available literature, for instance this scholarpedia article on the Unruh effect by Fulling and Matsas.

- This is the only exercise of this week that may require some calculation longer than one line. You can exploit the formulas (27)-(30) in section 9.3 of the lecture notes based on the trace anomaly. The only difference to these formulas is that you have to fix the integration constant t_- in (29) (and the corresponding quantity t_+ for the ingoing flux) differently from the lecture notes in order to ensure no radiation at $\mathscr{I}^\pm$. Prove that regularity of the stress-energy tensor in Kruskal coordinates implies that the fluxes in conformal coordinates, $T_{\pm\pm}$, need to vanish at the horizon and verify whether or not this is true for the Boulware vacuum.
  Further notes: It is sufficient to consider the two-dimensional part of the Schwarzschild metric for this exercise. Regarding Kruskal coordinates, recall the coordinate transformation (8.66) and (8.67) of Black Holes I between conformal and Kruskal coordinates. Finally, remember that the components of $T_{\mu\nu}$ transform with two Jacobian factors when doing a change of coordinates. The actual new calculation you need to do is then not longer than one line when you take all these hints into account.