

Black Holes II — Exercise sheet 2

(12.1) Penrose diagram for Robinson–Bertotti

We derived last semester (see exercise 9.1) the near horizon limit geometry of the extremal Reissner–Nordström black hole, which is known as Robinson–Bertotti geometry. Its line element is given by

$$ds^2 = -\lambda^2 dt^2 + Q^2 \frac{d\lambda^2}{\lambda^2} + Q^2 d\Omega_{S^2}^2$$

where Q is a constant (the charge) and $d\Omega_{S^2}^2$ is the line-element of the round S^2 . Show that the singularity at $\lambda = 0$ is merely a coordinate singularity. Show further that $\lambda = 0$ is a degenerate Killing horizon with respect to ∂_t . Finally, obtain the maximal analytic extension of the Robinson–Bertotti metric and deduce its Penrose diagram.

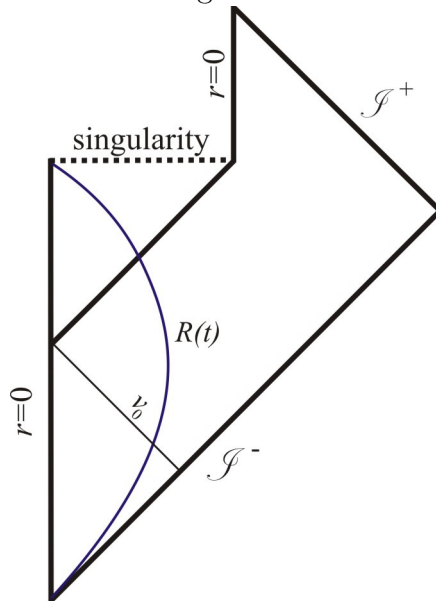
(12.2) Inventing Penrose diagrams

Draw (2-dimensional) Penrose diagrams for spacetimes with the following properties:

- Asymptotically flat, event horizon, no singularity
- Asymptotically flat, as many Killing horizons as possible, no Cauchy horizon
- Asymptotically flat, no event horizon, singularity
- Asymptotically flat, two non-extremal and one extremal Killing horizon
- Asymptotically flat, at least one Killing horizon, no singularity, no event horizon

(12.3) Penrose diagram of semi-classically evaporating black hole

What is wrong with the Penrose diagram below?



Hints:

- For the first two questions the coordinate transformation $u = t + Q/\lambda$, $v = t - Q/\lambda$ is helpful. For the final task the coordinate transformation $u = \tan(U/2)$, $v = -\cot(V/2)$ is convenient.
- Follow the algorithm explained during the lectures: start with the asymptotically flat region and “design” an Eddington–Finkelstein patch such that all requirements of the sub-exercise are met. Then, if possible, glue together copies of this Eddington–Finkelstein patch (and/or flipped versions thereof).
- Consider the domain of dependence of various $t = \text{const.}$ hypersurfaces. Are there Cauchy hypersurfaces? Is spacetime globally hyperbolic? (Why) should we care about these questions?

These exercises are due on March 17th 2020.