Daniel Grumiller May 19th 2021

Gravity and holography in lower dimensions II

(10.1) **Flat space holography in two bulk dimensions**
Consider a 2d dilaton gravity model with vanishing kinetic potential $U(X) = 0$ and constant self-interaction potential $V(X) = \Lambda$. After showing that all classical solutions are locally Ricci-flat, derive the asymptotic Killing vectors for the boundary conditions

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\big(\mathcal{P}(u)\,r + \mathcal{T}(u)\big)\,\mathrm{d}u^2$$

where $\delta\mathcal{P} \neq 0 \neq \mathcal{T}$ and check how the two state-dependent functions $\mathcal{P}$ and $\mathcal{T}$ transform under these asymptotic Killing vectors.

(10.2) **Flat space holography in three bulk dimensions**
Take a scaling limit of a rotating BTZ black hole metric

$$\mathrm{d}s^2_{\text{BTZ}} = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}\,\mathrm{d}t^2 + \frac{\ell^2 r^2\,\mathrm{d}r^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{r_+ r_-}{\ell r^2}\,\mathrm{d}t\right)^2$$

where you keep fixed the inner horizon and rescale the outer horizon $r_+ = \ell\hat{r}_+$ with the AdS-radius ℓ. Then take the limit $\ell \to \infty$ and study the resulting metric. In particular, show that this metric is Ricci flat (and hence solves flat space Einstein gravity) and construct the associated Penrose diagram (or rather, a 2d slice thereof). Are these geometries stationary in the asymptotic region? Do they describe black holes? [And if not, what else do they describe?]

(10.3) **Flat space holography in arbitrary dimensions?**
There are more than 16,000 papers quoting Maldacena's seminal work on AdS/CFT. Perhaps it is possible take the limit of infinite AdS radius (like in the previous exercise) to all these papers, thereby establishing a flat space holographic dictionary. Discuss the possible prospects and challenges for such a research program.

These exercises are due on June 8th 2021.

Hints/comments:

- You should find the asymptotic Killing vectors $\xi = \epsilon(u)\,\partial_u - (\epsilon'(u)r + \eta(u))\partial_r$ and the transformation laws $\delta_\xi \mathcal{P} = \epsilon\mathcal{P}' + \epsilon'\mathcal{P} + \epsilon''$ and $\delta_\xi \mathcal{T} = \epsilon\mathcal{T}' + 2\epsilon'\mathcal{T} + \eta' - \eta\mathcal{P}$.

- If you recall how ℓ entered in the equations of motion it is straightforward to show that your limiting metric must obey $R_{\mu\nu} = 0$, without actually having to calculate the Ricci tensor (though doing so explicitly also works, of course). If you additionally recall the Penrose diagram of BTZ you can again make a shortcut, just by declaring the locus $r = r_+$ to be your asymptotic boundary and omitting the regions where $r > r_+$ from your diagram (an explicit construction of the Penrose diagram using conformal compactification also works). Regarding the final questions simply calculate the asymptotic norm of the Killing vector ∂_t and make sure to remember the precise definition of a black hole.

- I leave this to you, but let me add five random remarks (that you need not consider, but you can).
  1. The conformal group is finite dimensional in higher dimensions, while the BMS group is infinite dimensional in any dimension.
  2. Taking the naive limit $\ell \to \infty$ in a Fefferman–Graham expansion produces nonsense, but you can exploit diffeos before any limits.
  3. There are different ways you might think about the dual field theory, depending on where you want to put it ($\mathscr{I}^+$, $\mathscr{I}^-$, i^0, i^+, and/or i^-)
  4. In asymptotically flat spacetimes we do already have a nice set of observables, namely S-matrix elements.
  5. Flat space is not just the limit of AdS for vanishing cosmological constant, but also of dS.