Soft Heisenberg hair

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based on work w. H. Afshar, S. Detournay,
W. Merbis, (B. Oblak), A. Perez,
D. Tempo, R. Troncoso, [S. Sheikh-Jabbari]

Simple punchline

Heisenberg algebra

$$[X_n, P_m] = i\,\delta_{n,m}$$

fundamental not only in quantum mechanics
but also in near horizon physics

Outline

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Motivation

Black hole microstates

Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

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- ▶ Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

  warped CFT: Detournay, Hartman, Hofman '12
  Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- ▶ Main idea: consider near horizon symmetries for non-extremal horizons

  Extremal horizons in 3d: see Li, Lucietti '14

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- ▶ Generalizations in $2+1$ gravity/gravity-like theories (Galilean CFT, warped CFT, ...)
- ▶ Main idea: consider near horizon symmetries for non-extremal horizons
- ▶ Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho \ \mathrm{d}v^2 + 2 \, \mathrm{d}v \, \mathrm{d}\rho + \gamma^2 \ \mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ▶ ρ: radial direction ($\rho = 0$ is horizon)
- ▶ $\varphi \sim \varphi + 2\pi$: angular direction
- ▶ v: (advanced) time

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of Rindler metric

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$$v \sim v + 2\pi L$$

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suggestion in 1511.08687

We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^{\pm} \wedge \mathrm{d}A^{\pm} + \tfrac{2}{3} A^{\pm} \wedge A^{\pm} \wedge A^{\pm} \rangle$$

  with $sl(2)$ connections $A^{\pm}$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

Outline

Diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho) \left(\mathrm{d} + \mathfrak{a}_{\pm}(x^0, x^1) \right) b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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▶ Standard AdS$_3$ approach: highest weight gauge

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$sl(2)$: $[L_n, L_m] = (n - m) L_{n+m}, \quad n, m = -1, 0, 1$

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▶ For near horizon purposes diagonal gauge useful:

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- For near horizon purposes diagonal gauge useful:

$$\mathfrak{a} \sim \mathcal{J}(x^0,\,x^1)\,L_0$$

- Precise boundary conditions (ζ: chemical potential):

$$\mathfrak{a} = \left(\mathcal{J}\,\mathrm{d}\varphi + \zeta\,\mathrm{d}v\right)L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\,\mathrm{d}\varphi\,L_0$$

  and $b = \exp\left(\frac{1}{\zeta}L_+\right) \cdot \exp\left(\frac{\rho}{2}L_-\right)$. (assume constant ζ for simplicity)

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Using

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yields ($f := 1 + \rho/(2a)$)

$$\begin{aligned}
\mathrm{d}s^2 = -2a\rho f \, \mathrm{d}v^2 + 2 \, \mathrm{d}v \, \mathrm{d}\rho - 2\omega a^{-1} \, \mathrm{d}\varphi \, \mathrm{d}\rho \\
+ 4\omega \rho f \, \mathrm{d}v \, \mathrm{d}\varphi + \left[\gamma^2 + \tfrac{2\rho}{a} f (\gamma^2 - \omega^2) \right] \mathrm{d}\varphi^2
\end{aligned}$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \mathrm{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm \partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

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Comments:

► Recover desired near horizon metric

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- Rindler acceleration a indeed state-independent

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- ▶ Recover desired near horizon metric
- ▶ Rindler acceleration a indeed state-independent
- ▶ Two state-dependent functions (γ, ω) as "usual"[1] in 3d gravity

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[1] However, see 1608.01308

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- Recover desired near horizon metric
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- Two state-dependent functions (γ, ω) as "usual" in 3d gravity
- $\gamma = \gamma(\varphi)$: "black flower"

Canonical boundary charges

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- Zero mode charges: mass and angular momentum

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Background independent result for Chern–Simons yields

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- Finite
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> Meaningful near horizon boundary conditions and non-trivial theory!

Outline

Near horizon symmetry algebra

▶ Near horizon symmetry algebra = all near horizon boundary
  conditions preserving trafos, modulo trivial gauge trafos

Most general trafo

$$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

that preserves our boundary conditions for constant ζ given by

$$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

with

$$\partial_v \eta = 0$$

implying

$$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

Near horizon symmetry algebra

- ▶ Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos
- ▶ Expand charges in Fourier modes

$$J_n^\pm = \frac{k}{4\pi} \oint \mathrm{d}\varphi\, e^{in\varphi} \mathcal{J}^\pm(\varphi)$$

What should we expect?

- ▶ Virasoro? (spacetime is locally AdS$_3$)
- ▶ BMS$_3$? (Rindler boundary similar to scri)
- ▶ warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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$$\left[J_n^{\pm}, \, J_m^{\pm} \right] = \pm \tfrac{1}{2} kn \delta_{n+m,\,0} \qquad \left[J_n^{+}, \, J_m^{-} \right] = 0$$

Two $\hat{u}(1)$ current algebras with non-zero levels

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- Much simpler than CFT_2, warped CFT_2, Galilean CFT_2, etc.

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  Two $\hat{u}(1)$ current algebras with non-zero levels

- Much simpler than CFT_2, warped CFT_2, Galilean CFT_2, etc.
- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \frac{i}{kn}\left(J_{-n}^+ + J_{-n}^-\right) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

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$$H := Q[\epsilon^\pm|_{\partial_v}] = a P_0$$

commutes with all generators of algebra

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▶ Same conclusion true for descendants of any state!

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> Soft hair = zero energy excitations on horizon

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$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

calculated directly in Chern–Sons formulation

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Before addressing microstates consider map to aymptotic variables

Map to asymptotic variables

- Usual asymptotic AdS$_3$ connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1} (\mathrm{d} + \hat{\mathfrak{a}}) \hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2} \mathcal{L} L_-$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \left(\tfrac{1}{2} \mu'' - \tfrac{1}{2} \mathcal{L} \mu \right) L_-$$

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$$g = \exp\left(xL_+\right)\cdot\exp\left(-\tfrac{1}{2}\mathcal{J}L_-\right)$$

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- Asymptotic charges: twisted Sugawara construction with near horizon charges

$$\mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

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$$\hat{A} = \hat{b}^{-1}\left(\mathrm{d}+\hat{\mathfrak{a}}\right)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

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- Gauge trafo $\hat{\mathfrak{a}} = g^{-1}\left(\mathrm{d}+\mathfrak{a}\right)g$ with

$$g = \exp\left(x L_+\right) \cdot \exp\left(-\tfrac{1}{2}\mathcal{J}L_-\right)$$

  where $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}x = 1$

- Near horizon chemical potential transforms into combination of asymptotic charge and chemical potential!

$$\mu' - \mathcal{J}\mu = -\zeta$$

- Asymptotic charges: twisted Sugawara construction with near horizon charges

$$\mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

▶ Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi} \oint \mathrm{d}\varphi \, \varepsilon \, \delta \mathcal{L} = -\frac{k}{4\pi} \oint \mathrm{d}\varphi \, \eta \, \delta \mathcal{J}$$

Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ

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$$\mu \mu'' - \tfrac{1}{2} \mu'^{\,2} - \mu^2 \mathcal{L} = -2\pi^2/\beta^2$$

Solved automatically from map to asymptotic observables; reminder:

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Near horizon boundary conditions natural for near horizon observer

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p \in \mathbb{Z}} J_{n-p} J_p + ikn J_n$$

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$$[L_n, L_m] = (n-m)L_{n+m} + \tfrac{1}{2} k\, n^3\, \delta_{n+m,\, 0}$$

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$$S_{\mathrm{Cardy}} = 2\pi\sqrt{kL_0^+} + 2\pi\sqrt{kL_0^-} = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N} = S_{\mathrm{BH}}$$

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Precise numerical factor in twist term crucial for correct results

---

Warped CFT counting

- Map near horizon algebra $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p} K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

$$[Y_n, Y_m] = (n - m)Y_{n+m}$$
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- Assuming $J^{\mathrm{vac}} = 0$ yields

$$S = \beta H = S_{\mathrm{BH}}$$

Hamiltonian H is product of BH entropy and Unruh temperature

Interpretation

> How do we interpret fact that Virasoro counting, warped counting,
> BMS counting, twisted warped counting ...
> all work and give the same entropy?

(My) Interpretation
Fineprint: these are just words

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- ▶ Dual field theory is simultaneously CFT, warped CFT, etc.!
- ▶ Should be possible to directly use Heisenberg for state counting

Outline

Comparison to related approaches

- Brown, Henneaux '86

  Our boundary conditions differ from Brown–Henneaux — their chemical potentials depend on our charges and chemical potentials!

  Virasoro composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino `1511.08687`
  - Observed already $H = T S_{\mathrm{BH}}$
  - Changing our bc's to

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \tfrac{2\rho}{a}(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2 + \mathcal{O}(\rho^2)$$

  yields AKVs

$$\xi = T(\varphi)\partial_v + Y(\varphi)\partial_\varphi + \mathcal{O}(\rho^3)$$

  - Up to subleading terms same AKVs as DGGP

  But: T and Y state-dependent for our boundary conditions!

Comment: map to Brown–Henneaux variables requires second chemical potential, not just Rindler acceleration!

  Warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

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> Rindler acceleration state-dependent in that approach

> Twisted warped CFT algebra composite in terms of Heisenberg algebra

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  - We constructed explicitly gravitational soft hair
  - We find no soft hair contribution to black hole entropy
  - BMS_3 follows from Sugawara-like construction from Heisenberg algebra

  > BMS algebra (supertranslations + superrotation) composite in terms of near horizon Heisenberg algebra

Comparison to related approaches

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- Comment on complementarity:

    - Asymptotic Virasoro algebra composite from near horizon perspective
    - Same physics described naturally in different variables for asymptotic and near horizon observers
    - In particular, asymptotic chemical potentials depend on near horizon charges and chemical potentials

Elaborations and generalizations

- More on dual field theory — to be done

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- ▶ More on dual field theory — to be done
- ▶ Technical elaborations — to be done

    e.g. well-defined variational principle in metric formulation; comparison with Stephane Detournay, DG, Friedrich Schöller, Joan Simón '14

Elaborations and generalizations

- More on dual field theory — to be done
- Technical elaborations — to be done
- Flat space
  - Similar story works!
  - Get centerless BMS_3 as composite algebra from Heisenberg algebra!
  - Soft hairy flat space cosmologies
  - Asymptotic chemical potentials again depend on near horizon charges and chemical potentials
  - Obtain again Bekenstein–Hawking entropy with no soft hair contribution

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- Higher spins — with Alfredo Perez, Stefan Prohazka, David Tempo, Ricardo Troncoso '16: similar story works!

  So far: higher spins in AdS_3; generalizations to higher spins in non-AdS backgrounds?

  see e.g. work with Mirah Gary, Radoslav Rashkov '12 and with Veronika Breunhölder, Mirah Gary, Stefan Prohazka and Soo-Jong Rey '15

  for flat space higher spins see work with Hamid Afshar, Arjun Bagchi, Reza Fareghbal, Jan Rosseel '13 and independently work by Hernan Gonzalez, Javier Matulich, Miguel Pino and Ricardo Troncoso '13

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- ▶ 4d — Does it work? Is there soft Heisenberg hair? Is BMS_4 composite? What are near horizon symmetries?

  - ▶ Algebraic indication: near horizon symmetry algebra by DGGP '16 again compositive in terms of infinitely many Heisenberg algebras!
  - ▶ Non-trivial indication that Heisenberg story could work in 4d as well
  - ▶ see paper with Hamid Afshar and Shahin Sheikh-Jabbari '16

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> Near horizon symmetries shed new light on soft hair, microstate counting and complementarity

Thanks for your attention!

📄 H. Afshar, D. Grumiller and M.M. Sheikh-Jabbari
"Near Horizon Soft Hairs as Microstates of 3D Black Holes,"
1607.00009.

📕 D. Grumiller, A. Perez, S. Prohazka, D. Tempo and R. Troncoso
"Higher Spin Black Holes with Soft Hair," JHEP, in print;
1607.05360

📕 **H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso**
"Soft Heisenberg hair on black holes in three dimensions,"
Phys.Rev.D [R] (2016) 101503; 1603.04824**.**

📕 H. Afshar, S. Detournay, D. Grumiller and B. Oblak
"Near-Horizon Geometry and Warped Conformal Symmetry,"
JHEP **1603** (2016) 187; 1512.08233.

Bonus level: exact metric with generic chemical potentials

Our bc's for the connection $A^\pm = b_\pm^{-1}(\rho)\left(\mathrm{d} + \mathfrak{a}_\pm(x^0,\, x^1)\right)b_\pm(\rho)$ with

$$\mathfrak{a}_\pm = \left(\mathcal{J}_\pm\, \mathrm{d}\varphi + \zeta^\pm\, \mathrm{d}v\right) L_0$$

and $b_\pm = \exp\left(\frac{1}{\zeta^\pm} L_+\right)\cdot \exp\left(\frac{\rho}{2} L_-\right)$ lead to the metric

$$\mathrm{d}s^2 = \tfrac{1}{2}\left\langle \left(A_\mu^+ - A_\mu^-\right)\left(A_\nu^+ - A_\nu^-\right)\right\rangle \mathrm{d}x^\mu\, \mathrm{d}x^\nu$$

$$= \left(-\frac{(\zeta^{+2}+\partial_v\zeta^+)(\zeta^{-2}+\partial_v\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2 + \frac{\zeta^{+3}\zeta^{-2}+\zeta^{+2}\zeta^{-3}+\partial_v\zeta^+\zeta^{-3}+\zeta^{+3}\partial_v\zeta^-}{\zeta^{+2}\zeta^{-2}}\rho + \tfrac{1}{4}(\zeta^- - \zeta^+)^2\right)\mathrm{d}v^2$$

$$+ \left(\frac{(-\zeta^{+2}-\partial_v\zeta^+)\partial_\varphi\zeta^- + (-\zeta^{-2}-\partial_v\zeta^-)\partial_\varphi\zeta^+ - \mathcal{J}_+\zeta^+\partial_v\zeta^- + \zeta^-(\mathcal{J}_-\zeta^{+2}-\mathcal{J}_+\zeta^+\zeta^- + \mathcal{J}_-\partial_v\zeta^+)}{2\zeta^{+2}\zeta^{-2}}\rho^2\right.$$

$$+ \frac{\partial_\varphi\zeta^-\zeta^{+3}+\partial_\varphi\zeta^+\zeta^{-3}+\mathcal{J}_+\zeta^{+2}\partial_v\zeta^- - \zeta^-\left(\mathcal{J}_-\partial_v\zeta^+\zeta^- + \zeta^+(\zeta^- + \zeta^+)(\zeta^+\mathcal{J}_- - \zeta^-\mathcal{J}_+)\right)}{2\zeta^{+2}\zeta^{-2}}\rho$$

$$\left.-\tfrac{1}{4}(\zeta^- - \zeta^+)(\mathcal{J}_- + \mathcal{J}_+)\right)\mathrm{d}v\,\mathrm{d}\varphi + \left(1 + \frac{\partial_v\zeta^-\zeta^{+2}+\partial_v\zeta^+\zeta^{-2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}v\,\mathrm{d}\rho$$

$$+ \left(\frac{(\mathcal{J}_+\zeta^+ + \partial_\varphi\zeta_+)(\mathcal{J}_-\zeta^- - \partial_\varphi\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2 + \frac{\mathcal{J}_+\partial_\varphi\zeta^-\zeta^{+2} - \zeta^-\mathcal{J}_-(\zeta^-\partial_\varphi\zeta^+ + \mathcal{J}_+\zeta^+(\zeta^- + \zeta^+))}{\zeta^{+2}\zeta^{-2}}\rho\right.$$

$$\left.+\tfrac{1}{4}(\zeta^- + \zeta^+)^2\right)\mathrm{d}\varphi^2 + \left(\frac{\mathcal{J}_+\zeta^+\zeta^{-2} - \mathcal{J}_-\zeta^{+2}\zeta^- + \partial_\varphi\zeta^+\zeta^{-2} + \partial_\varphi\zeta^-\zeta^{+2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}\varphi\,\mathrm{d}r$$