Gravity in two dimensions

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Outline

Why lower-dimensional gravity?

Which 2D theory?

Quantum dilaton gravity with matter

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The Holy Grail of theoretical physics

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- Experimental signatures? Data?

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
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- 3D: lowest dimension exhibiting BHs and gravitons
- Simplest gravitational theories with BHs and gravitons in 3D

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Quantum dilaton gravity with matter

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us start with the simplest attempt. `Einstein-Hilbert` action in 2 dimensions:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^2x \sqrt{|g|}\, R = \frac{1}{2G}\,(1-\gamma)$$

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- Action is topological
- No equations of motion
- Formal counting of number of gravitons: -1

[in D dimensions Einstein gravity has $D(D-3)/2$ graviton polarizations]

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- Mann: limit $\epsilon \to 0$ should be possible and lead to 2D dilaton gravity
- DG, Jackiw: limit $\epsilon \to 0$ (with $G(\epsilon \to 0) \to 0$) yields Liouville gravity

$$\lim_{\epsilon \to 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{|g|}\, \left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

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Result of attempt 1:

A specific 2D dilaton gravity model

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

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Result of attempt 2:

A specific 2D dilaton gravity model

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \mathrm{d}x^\mu \, \mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \mathrm{d}x^\alpha \, \mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \mathrm{d}\Omega^2_{S_{D-2}} \,,$$

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Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g} \, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \big((\nabla\phi)^2 - 1 \big) \Big]$$

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Cosmetic redefinition $X \propto (\lambda\phi)^{D-2}$:

$$I_{EH} = \frac{1}{16\pi \, G_2} \int \mathrm{d}^2 x \sqrt{-g} \Big[X R + \frac{D-3}{(D-2)X} (\nabla X)^2 - \lambda^2 X^{(D-4)/(D-2)} \Big]$$

Result of attempt 3:

A specific class of 2D dilaton gravity models

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

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- Example: `Katanaev-Volovich` model (Poincare gauge theory)

$$I_{\rm KV} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- `Kummer, Schwarz`: bring into first order form:

$$I_{\rm KV} \sim \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega\wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a\wedge e^b\,(\alpha X^a X_a + \beta X^2)\Big]$$

- Use same algorithm as before to convert into second order action:

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A specific 2D dilaton gravity model

Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|} \left[g_{\mu\nu} h^{ij} \partial_i x^\mu \partial_j x^\nu + \alpha' \phi \mathcal{R} + \dots \right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$
$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{-g} \Big[XR + \frac{1}{X}\,(\nabla X)^2 - 4b^2 \Big]$$

where $X = e^{-2\phi}$

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Selected List of Models

Black holes in $(A)dS$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten Black Hole (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	$-2b^2$
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. Black Hole attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A\sinh X$
17. 2D type 0A string Black Hole	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string Black Hole (2005)	lengthy	lengthy

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{|g|} \left[XR - U(X)(\nabla X)^2 - V(X) \right]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\;\; [XK - S(X)] + I^{(m)}$$

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$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y} \int^X V(y) e^{\int^y U(z)\,\mathrm{d}z}\,\mathrm{d}y$$

and guarantees well-defined variational principle $\delta I = 0$

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- Interesting option: couple 2D dilaton gravity to matter

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Implement the following algorithm:

Exact path integral quantization of 2D dilaton gravity

Overview of the quantization procedure

Implement the following algorithm:

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- Calculate Feynman rules

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- Do the BRST gymnastics
- Integrate out geometry exactly!
- Obtain non-local non-polynomial matter action
- Treat matter perturbatively
- Calculate Feynman rules
- Reconstruct intermediate geometry (Virtual BHs)

Implement the following algorithm:

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- Reconstruct intermediate geometry (Virtual BHs)
- Calculate S-matrix or corrections to classical quantities (like specific heat)

Non-minimally coupled matter

Prominent example: Einstein-massless Klein-Gordon model (Choptuik)

- no matter: integrability, no scattering, no propagating physical modes
- with matter: no integrability in general, scattering, critical collapse

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Massless scalar field S:

$$I^{(m)} = \int \mathrm{d}^2x \sqrt{-g} F(X) (\nabla S)^2$$

- minimal coupling: $F = \text{const.}$
- non-minimal coupling otherwise
- spherical reduction: $F \propto X$

Non-perturbative path integral quantization

Integrating out geometry exactly

- constraint analysis $\{G^i(x), G^j(x')\} = G^k C_k{}^{ij} \delta(x - x')$
- BRST charge $\Omega = c^i G_i + c^i c^j C_{ij}{}^k p_k$ (ghosts c^i, p_k)
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integrating ghost sector yields

$$Z[\text{sources}] = \int \mathcal{D}f \delta \left(f + i\delta/\delta j_{e_1^+} \right) \tilde{Z}[f, \text{sources}]$$

with $(\tilde{S} = S\sqrt{f})$

$$\tilde{Z}[f, \text{sources}] = \int \mathcal{D}\tilde{S}\mathcal{D}(\omega, e^a, X, X^a) \det \Delta_{F.P.} \exp i(I_{g.f.} + \text{sources})$$

Can integrate over all fields except matter non-perturbatively!

Non-local effective theory

Convert local gravity theory with matter into non-local matter theory without gravity

Generating functional for Green functions ($F = 1$):

$$\tilde{Z}[f, \text{sources}] = \int \mathcal{D}\tilde{S} \exp i \int (\mathcal{L}^k + \mathcal{L}^v + \mathcal{L}^s) d^2x$$

$$\mathcal{L}^k = \partial_0 S \partial_1 S - E_1^- (\partial_0 S)^2 \,, \; \mathcal{L}^v = -w'(\hat{X}) \,, \; \mathcal{L}^s = \sigma S + j_{e_1^+} \hat{E}_1^+ + \ldots ,$$

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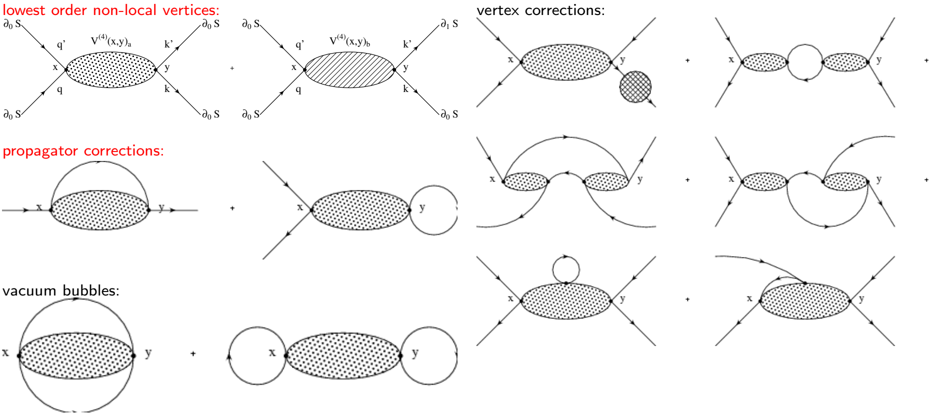
$$\tilde{S} = S f^{1/2} \,,\ \hat{E}_1^+ = e^{Q(\hat{X})} \,,\ \hat{X} = \underbrace{a + b x^0}_{X} + \underbrace{\partial_0^{-2}(\partial_0 S)^2}_{\text{non-local}} + \dots \,,\ a = 0 \,,\ b = 1 \,,$$

$$E_1^- = w(X) + M \,, \quad \hat{E}_1^+ = e^{Q(X)} + e^{Q(X)} U(X) \partial_0^{-2} (\partial_0 S)^2 + \dots$$

$$\int \mathcal{D}\tilde{S} \exp i \int \mathcal{L}^k = \exp \underbrace{\left(i/96\pi \int_x \int_y f R_x \Box^{-1}_{xy} R_y \right)}_{\text{Polyakov}}$$

Red: geometry, Magenta: matter, Blue: boundary conditions

Some Feynman diagrams



- so far: calculated only lowest order vertices and propagator corrections
- partial resummations possible (similar to Bethe-Salpeter)?
- non-local loops vanish to this order

S-matrix for s-wave gravitational scattering

Quantizing the Einstein-massless-Klein-Gordon model

ingoing s-waves $q = \alpha E, q' = (1 - \alpha)E$ interact and scatter into outgoing s-waves $k = \beta E, k' = (1 - \beta)E$

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Quantizing the Einstein-massless-Klein-Gordon model

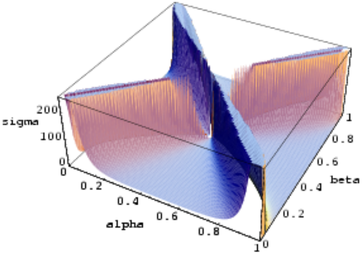
ingoing s-waves $q = \alpha E, q' = (1-\alpha)E$ interact and scatter into outgoing s-waves $k = \beta E, k' = (1-\beta)E$

$$T(q,q';k,k') \propto \tilde{T}\delta(k+k'-q-q')/|kk'qq'|^{3/2} \tag{1a}$$

with $\Pi = (k+k')(k-q)(k'-q)$ and

$$\tilde{T} = \Pi \ln \frac{\Pi^2}{E^6} + \frac{1}{\Pi}\sum_p p^2 \ln \frac{p^2}{E^2} \cdot \left(3kk'qq' - \frac{1}{2}\sum_{r\neq p}\sum_{s\neq r,p} r^2 s^2\right) \tag{1b}$$

Plot of cross-section



- result finite and simple
- monomial scaling with E
- forward scattering poles $\Pi = 0$
- decay of s-waves possible
- not understood why so simple!
  (intermediate results vastly more complicated)

Other selected successes of (quantum) dilaton gravity with/without matter

- Gravity as non-abelian gauge theory Jackiw, Teitelboim '84
- Black holes in string theory Witten '91
- Black hole evaporation Callan, Giddings, Harvey, Strominger '92
- Gravity as non-linear gauge theory Ikeda, Izawa '93
- Dirac quantization Louis-Martinez, Gegenberg, Kunstatter '94
- All classical solutions Klösch, Strobl '96 –'98
- Virtual black holes DG, Kummer, Vassilevich '00
- Unitary S-matrix DG, Kummer, Vassilevich '01
- Quantum corrected specific heat DG, Kummer, Vassilevich '03
- Liouville Field Theory Nakayama '04
- Duality DG, Jackiw '06
- Holographic renormalization DG, McNees '07
- Central charge in AdS_2 Hartman, Strominger '08
- AdS_2 holography Castro, DG, Larsen, McNees '08
- Model for gravity at large distances DG '10
- Quantization of cosmological constant? Govaerts, Zonetti '11

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Summary — Thank you for your attention!

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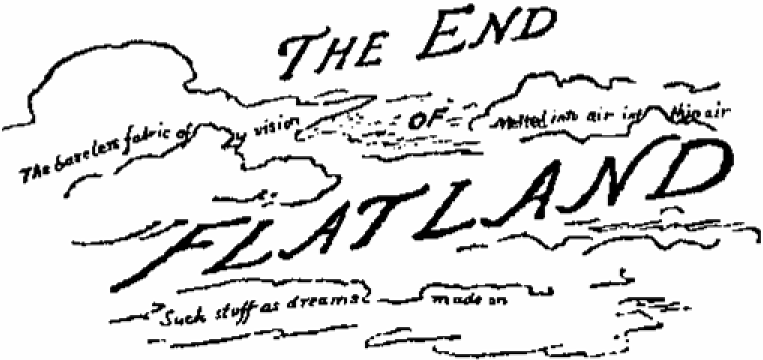

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- D. Grumiller, W. Kummer, and D. Vassilevich, "Dilaton gravity in two dimensions," *Phys. Rept.* **369** (2002) 327–429, `hep-th/0204253`.
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- J. Govaerts, S. Zonetti, "Quantized cosmological constant in 1+1 dimensional quantum gravity with coupled scalar matter," `1102.4957`.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\,F^2\right]$$

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- δA EOM: $\nabla_\mu F^{\mu\nu} = 0$ $\Rightarrow$ $E = \text{constant}$
- δg EOM: complicated for non-constant dilaton...

$$\nabla_\mu\nabla_\nu e^{-2\phi} - g_{\mu\nu}\,\nabla^2 e^{-2\phi} + \frac{4}{L^2}\,e^{-2\phi}\,g_{\mu\nu} + \frac{L^2}{2}\,F_\mu{}^\lambda\,F_{\nu\lambda} - \frac{L^2}{8}\,g_{\mu\nu}\,F^2 = 0$$

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- δg EOM: ...but simple for constant dilaton: $e^{-2\phi} = \frac{L^4}{4}\, E^2$

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Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

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- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t \xi + \xi \partial_t T_{tt} - \frac{c}{24\pi} L \partial_t^3 \xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L \partial_t \lambda$)

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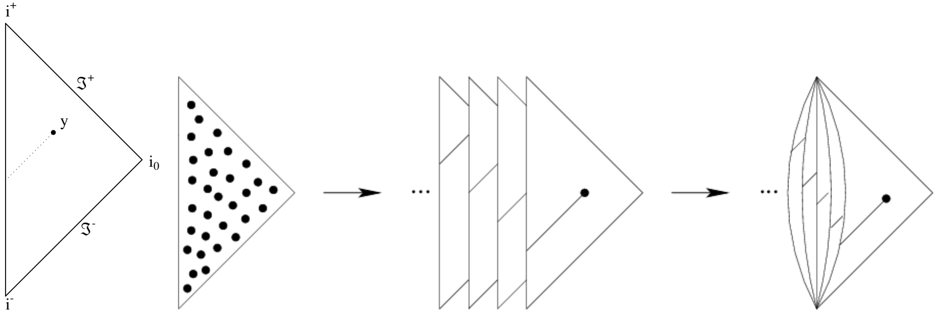
- Positive central charge only for negative coupling constant α (CGLM)

$$\alpha < 0$$

Virtual black holes

Reconstruct geometry from matter

"Intermediate geometry" (caveat: off-shell!):



$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r + [1 - \underbrace{\delta(u-u_0)\theta(r_0-r)}_{\text{localized}}(2M/r + ar + d)]\,\mathrm{d}u^2$$

- Schwarzschild and Rindler terms
- nontrivial part localized
- geometry is non-local (depends on $r, u, \underbrace{r_0, u_0}$)