Flat space holography

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1402.3687, 1307.4768, 1305.2919, 1208.4372, 1208.1658

Statement of the main result

Hot flat space $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}s^2 = \pm\, \mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \; \mathrm{d}\varphi^2$$

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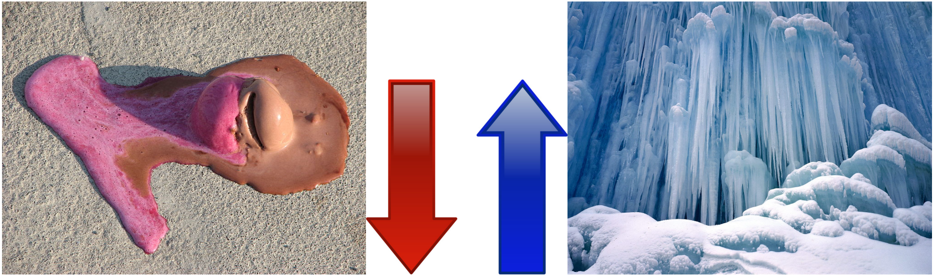

$$\mathrm{d}s^2 = \pm\,\mathrm{d}\tau^2 + \frac{(E\tau)^2\,\mathrm{d}x^2}{1+(E\tau)^2} + \big(1+(E\tau)^2\big)\Big(\mathrm{d}y + \frac{(E\tau)^2}{1+(E\tau)^2}\,\mathrm{d}x\Big)^2$$

Flat space cosmology $(y \sim y + 2\pi r_0)$

Bagchi, Detournay, Grumiller & Simon '13

Outline

Motivation: Gravity in lower dimensions

Review: AdS/CFT from a relativist's perspective

Developments: Flat space holography

Novel result: Cosmic phase transition

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Motivation for studying gravity in 2 and 3 dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics
  - Models should be as simple as possible, but not simpler
- Gauge/gravity duality + indirect physics applications
  - Deeper understanding of black hole holography
  - AdS_3/CFT_2 correspondence best understood
  - Quantum gravity via AdS/CFT
  - Applications to 2D condensed matter systems
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, Lifshitz, Schrödinger, non-relativistic or log CFTs, higher spin holography ...
  - Flat space holography
- Direct physics applications
  - Cosmic strings
  - Black hole analog systems in condensed matter physics
  - Effective theory for gravity at large distances

Outline

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle

   Example: Einstein gravity with Dirichlet boundary conditions

$$I = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{|g|}\,\big(R+\frac{2}{\ell^2}\big)$$

   with $\delta g = \text{fixed}$ at the boundary

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   Example: asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(e^{2\rho/\ell}\,\gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} + \dots\right)\,\mathrm{d}x^i\,\mathrm{d}x^j$$

   with $\delta\gamma^{(0)} = 0$ for $\rho \to \infty$

Holographic algorithm from gravity point of view

Universal recipe:

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3. Perform canonical analysis and check consistency of bc's
   Example: Brown–Henneaux analysis for 3D Einstein gravity

$$\{Q[\varepsilon],\, Q[\eta]\} = \delta_\varepsilon Q[\eta]$$

with

$$Q[\varepsilon] \sim \oint \mathrm{d}\varphi\, \mathcal{L}(\varphi)\varepsilon(\varphi)$$

and

$$\delta_\varepsilon \mathcal{L} = -\mathcal{L}\,\varepsilon - 2\mathcal{L}\,\varepsilon' - \frac{\ell}{16\pi G_N}\,\varepsilon'''$$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
   Example: Two copies of Virasoro algebra

$$[\mathcal{L}_n,\, \mathcal{L}_m] = (n-m)\,\mathcal{L}_{n+m} + \frac{c}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

   with Brown–Henneaux central charge

$$c = \frac{3\ell}{2G_N}$$

   Reminder: ASA = quotient algebra of asymptotic symmetries by 'trivial' asymptotic symmetries with zero canonical charges

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5. Improve to quantum ASA
   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey '10; Campoleoni, Pfenninger, Fredenhagen, Theisen '10)

$$[W_n,\, W_m] = \frac{16}{5c}\sum_p L_p L_{n+m-p} + \dots$$

   quantum ASA

$$[W_n,\, W_m] = \frac{16}{5c+22}\sum_p :L_p L_{n+m-p}: + \dots$$

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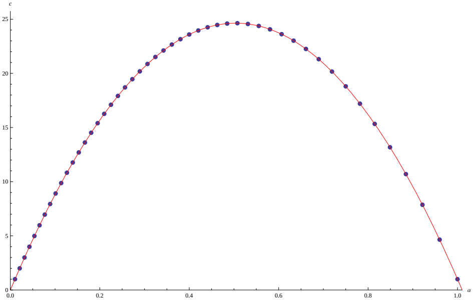
Example:



Afshar et al '12

Discrete set of Newton constant values compatible with unitarity (3D spin-N gravity in next-to-principal embedding)

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7. Identify/constrain dual field theory

   Example: Monster CFT in (flat space) chiral gravity

   Witten '07

   Li, Song & Strominger '08

   Bagchi, Detournay & Grumiller '12

$$Z(q) = J(q) = \frac{1}{q} + (1 + 196883)\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

Holographic algorithm from gravity point of view

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   Examples: too many!

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Goal of this talk:

Apply algorithm above to flat space holography in 3D gravity

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Flat space from contraction of AdS

Idea: take $\ell \to \infty$ limit of AdS results!

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- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
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  Ashtekar, Bicak & Schmidt '96

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  Ashtekar, Bicak & Schmidt '96
- Example where it does not work easily: boundary conditions!

Apply algorithm just described

1. Identify bulk theory and variational principle
   Topologically massive gravity with mixed boundary conditions

$$I = I_{\text{EH}} + \frac{1}{32\pi\, G\mu} \int \mathrm{d}^3x \sqrt{-g}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma} \big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau} \Gamma^\tau{}_{\nu\rho} \big)$$

   with $\delta g = \text{fixed}$ and $\delta K_L = \text{fixed}$ at the boundary
   Deser, Jackiw & Templeton '82

Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   asymptotically flat adapted to lightlike infinity $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}\bar{s}^2 = -\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,\mathrm{d}\varphi^2$$

$$\begin{aligned}
g_{uu} &= h_{uu} + O(\tfrac{1}{r}) \\
g_{ur} &= -1 + h_{ur}/r + O(\tfrac{1}{r^2}) \\
g_{u\varphi} &= h_{u\varphi} + O(\tfrac{1}{r}) \\
g_{rr} &= h_{rr}/r^2 + O(\tfrac{1}{r^3}) \\
g_{r\varphi} &= h_1(\varphi) + h_{r\varphi}/r + O(\tfrac{1}{r^2}) \\
g_{\varphi\varphi} &= r^2 + (h_2(\varphi) + u h_3(\varphi))r + O(1)
\end{aligned}$$

Barnich & Compere '06
Bagchi, Detournay & Grumiller '12

Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
   Obtain canonical boundary charges

$$Q_{M_n} = \frac{1}{16\pi G}\int \mathrm{d}\varphi\, e^{in\varphi}\,\big(h_{uu} + h_3\big)$$

$$\begin{aligned} Q_{L_n} &= \frac{1}{16\pi G\mu}\int \mathrm{d}\varphi\, e^{in\varphi}\,\big(h_{uu} + \partial_u h_{ur} + \tfrac{1}{2}\partial_u^2 h_{rr} + h_3\big) \\ &+ \frac{1}{16\pi G}\int \mathrm{d}\varphi\, e^{in\varphi}\,\big(inu h_{uu} + in h_{ur} + 2h_{u\varphi} + \partial_u h_{r\varphi} \\ &\qquad - (n^2 + h_3)h_1 - in h_2 - in\partial_\varphi h_1\big) \end{aligned}$$

Bagchi, Detournay & Grumiller '12

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$$[M_n,\, M_m] = 0$$

with central charges

$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

Note:

- $c_L = 0$ in Einstein gravity
- $c_M = 0$ in conformal Chern–Simons gravity ($\mu \to 0$, $\mu G = \frac{1}{8k}$)
  Flat space chiral gravity!
  Bagchi, Detournay & Grumiller '12

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   Trivial here

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6. Study unitary representations of quantum ASA
    - Straightforward in flat space chiral gravity
    - Difficult/impossible otherwise

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   Monster CFT in flat space chiral gravity
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But...:

What about non-perturbative states analogue to BTZ black holes?
Where/what are they in flat space (chiral) gravity?

Outline

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Developments: Flat space holography

Novel result: Cosmic phase transition

Flat space cosmologies (Cornalba & Costa '02)

- Start with BTZ in AdS:

$$\mathrm{d}s^2 = -\frac{(r^2-R_+^2)(r^2-r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2-R_+^2)(r^2-r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{R_+ r_-}{\ell r^2}\,\mathrm{d}t\right)^2$$

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- Consider region between the two horizons $r_- < r < R_+$
- Take the $\ell \to \infty$ limit (with $R_+ = \ell \hat{r}_+$ and $r_- = r_0$)

$$\mathrm{d}s^2 = \hat{r}_+^2\big(1 - \frac{r_0^2}{r^2}\big)\,\mathrm{d}t^2 - \frac{r^2\,\mathrm{d}r^2}{\hat{r}_+^2\,(r^2 - r_0^2)} + r^2\big(\mathrm{d}\varphi - \frac{\hat{r}_+ r_0}{r^2}\,\mathrm{d}t\big)^2$$

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- Go to Euclidean signature ($t = i\tau_E$, $\hat{r}_+ = -ir_+$)

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Question we want to address:

Is FSC or HFS the preferred Euclidean saddle?

Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

$$Z(T,\,\Omega)=\int\mathcal{D}g\,e^{-\Gamma[g]}=\sum_{g_c}e^{-\Gamma[g_c(T,\Omega)]}\times Z_{\text{fluct.}}$$

boundary conditions specified by temperature T and angular velocity Ω

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boundary conditions specified by temperature T and angular velocity Ω
Two Euclidean saddle points in same ensemble if

- same temperature T and angular velocity Ω
- obey flat space boundary conditions
- solutions without conical singularities

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HFS:

$$(\tau_E,\,\varphi)\sim(\tau_E+\beta,\,\varphi+\beta\Omega)\sim(\tau_E,\,\varphi+2\pi)$$

FSC:

$$(\tau_E,\,\varphi)\sim(\tau_E+\beta,\,\varphi+\beta\Omega)\sim(\tau_E,\,\varphi+2\pi)$$

Results

On-shell action (1/2 Gibbons–Hawking–York boundary term!):

$$\Gamma = -\frac{1}{16\pi G_N} \int \mathrm{d}^3x\sqrt{g}\, R - \frac{1}{16\pi G_N} \int \mathrm{d}^2x\sqrt{\gamma}\, K$$

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Free energy:

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

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Free energy:

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- $r_+ > 1$: FSC dominant saddle
- $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

Discussion and generalization

- Free energy of FSC: $F(T,\,\Omega) = -\frac{\pi T}{4G_N\Omega}$
- Entropy: $S = \frac{2\pi r_0}{4G_N}$ (BH area law)
- First law: $\mathrm{d}F = -S\,\mathrm{d}T - J\,\mathrm{d}\Omega$
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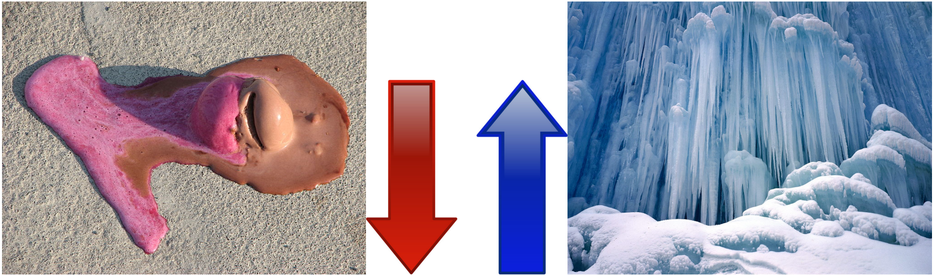
- Non-negative specific heat
- Generalizations: should be easy to consider NMG, GMG, ... in 3D
- Higher dimensions?

Summary of the main result

Hot flat space $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}s^2 = \mathrm{d}t^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$$



$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + \frac{(E\tau)^2\,\mathrm{d}x^2}{1+(E\tau)^2} + \big(1+(E\tau)^2\big)\Big(\mathrm{d}y + \frac{(E\tau)^2}{1+(E\tau)^2}\,\mathrm{d}x\Big)^2$$

Flat space cosmology $(y \sim y + 2\pi r_0)$

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Appendix: Coordinate transformation to Cornalba–Costa line-element

FSC in BTZ coordinates:

$$\mathrm{d}s^2 = \hat{r}_+^2 \left(1 - \frac{r_0^2}{r^2}\right) \mathrm{d}t^2 - \frac{r^2 \, \mathrm{d}r^2}{\hat{r}_+^2 \left(r^2 - r_0^2\right)} + r^2 \left(\mathrm{d}\varphi - \frac{\hat{r}_+ r_0}{r^2} \, \mathrm{d}t\right)^2$$

Coordinate trafo:

$$\hat{r}_+ t = -x$$
$$r_0 \varphi = x + y$$
$$(r/r_0)^2 = 1 + (E\tau)^2$$
$$E = \hat{r}_+ / r_0$$

FSC in CC coordinates:

$$\mathrm{d}s^2 = -\,\mathrm{d}\tau^2 + \frac{(E\tau)^2 \, \mathrm{d}x^2}{1 + (E\tau)^2} + \left(1 + (E\tau)^2\right) \left(\mathrm{d}y + \frac{(E\tau)^2}{1 + (E\tau)^2} \, \mathrm{d}x\right)^2$$