Flat Space Holography

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FWF based on work w. Afshar, Bagchi, Basu, Detournay, Fareghbal, Gary, Riegler, Rosseel, Salzer, Sarkar, Schöller, Simon, ...

Outline

Motivations

Holography basics

Flat space holography

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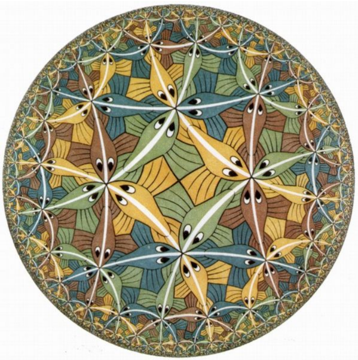
  - Holographic principle realized in Nature? (yes/no)



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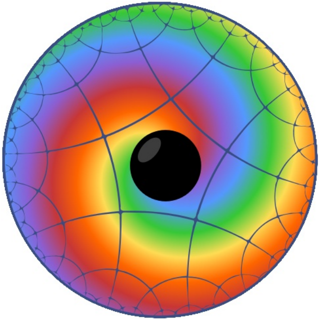

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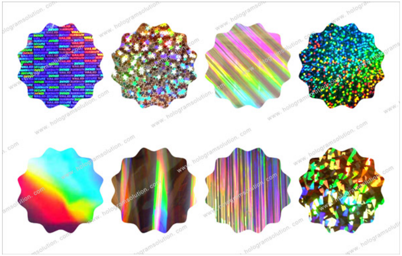

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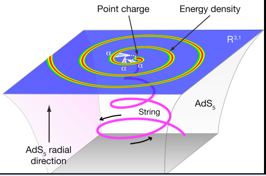
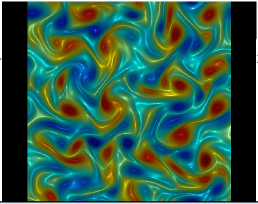
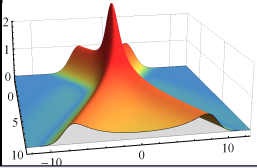
- Applications
  - Gauge gravity correspondence (plasmas, condensed matter, ...)







Specific motivation for 3D

Gravity in 3D is simpler than in higher dimensions

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Address these issues in 3D!

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Interesting dichotomy:

- Either dual field theory exists $\rightarrow$ useful toy model for quantum gravity
- Or gravitational theory needs UV completion (within string theory) $\rightarrow$ indication of inevitability of string theory

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This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding

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AdS_3 gravity

- Lowest dimension with black holes and (off-shell) gravitons

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Interesting generic constraints from CFT_2!
e.g. Hellerman '09, Hartman, Keller, Stoica '14

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Caveat: while there are many string compactifications with AdS_3 factor, applying holography just to AdS_3 factor does not capture everything!

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
   Example: Einstein gravity with Dirichlet boundary conditions
   $$I = -\frac{1}{16\pi G_N} \int \mathrm{d}^3x \sqrt{|g|} \left(R + \frac{2}{\ell^2}\right)$$
   with $\delta g = \text{fixed}$ at the boundary

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   Example: asymptotically AdS
   $$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \big(e^{2\rho/\ell}\,\gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} + \dots\big)\,\mathrm{d}x^i\,\mathrm{d}x^j$$
   with $\delta\gamma^{(0)} = 0$ for $\rho \to \infty$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
    - Find and classify all constraints
    - Construct canonical gauge generators
    - Add boundary terms and get (variation of) canonical charges
    - Check integrability of canonical charges
    - Check finiteness of canonical charges
    - Check conservation (in time) of canonical charges
    - Calculate Dirac bracket algebra of canonical charges

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Example: Brown–Henneaux analysis for 3D Einstein gravity

$$\{Q[\varepsilon],\, Q[\eta]\} = \delta_\varepsilon Q[\eta]$$

$$Q[\varepsilon] \sim \oint \mathrm{d}\varphi\, \mathcal{L}(\varphi)\varepsilon(\varphi)$$

$$\delta_\varepsilon \mathcal{L} = \mathcal{L}\,\varepsilon + 2\mathcal{L}\,\varepsilon' + \frac{\ell}{16\pi G_N}\,\varepsilon'''$$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
   Example: Two copies of Virasoro algebra

$$[\mathcal{L}_n,\, \mathcal{L}_m] = (n-m)\,\mathcal{L}_{n+m} + \frac{c}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

   with Brown–Henneaux central charge

$$c = \frac{3\ell}{2G_N}$$

   Reminder: ASA = quotient algebra of asymptotic symmetries by 'trivial' asymptotic symmetries with zero canonical charges

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5. Improve to quantum ASA

   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey '10; Campoleoni, Pfenninger, Fredenhagen, Theisen '10)

$$[W_n, W_m] = \frac{16}{5c} \sum_p L_p L_{n+m-p} + \dots$$

   quantum ASA

$$[W_n, W_m] = \frac{16}{5c+22} \sum_p : L_p L_{n+m-p} : + \dots$$

Holographic algorithm from gravity point of view

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1. Identify bulk theory and variational principle
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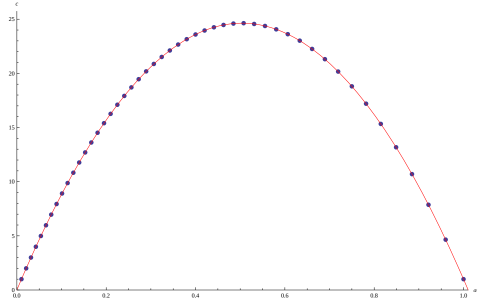
6. Study unitary representations of quantum ASA

Example:



Afshar et al '12
Discrete set of Newton constant values compatible with unitarity
(3D spin-N gravity in next-to-principal embedding)

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7. Identify/constrain dual field theory

   Example: Monster CFT in (flat space) chiral gravity

   Witten '07

   Li, Song & Strominger '08

   Bagchi, Detournay & DG '12

$$Z(q) = J(q) = \frac{1}{q} + (1 + 196883)\, q + \mathcal{O}(q^2)$$

   Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

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8. If unhappy with result go back to previous items and modify

   Examples: too many!

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Goal of this talk:

Apply algorithm above to flat space holography in 3D gravity theories

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$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$

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- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!

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Not in general! Must (also) work intrinsically in flat space!
Interesting example:

- unitarity of flat space quantum gravity

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> Many open issues in flat space holography!

Next few slides: mention a couple of recent results

Overview of selected recent results

- Applying algorithm just described to flat space theories

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- Unitarity of dual field theory
- Adding chemical potentials

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
   Topologically massive gravity with mixed boundary conditions

$$I = I_{\text{EH}} + \frac{1}{32\pi\, G\mu} \int \mathrm{d}^3x \sqrt{-g}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

   with $\delta g = \text{fixed}$ and $\delta K_L = \text{fixed}$ at the boundary
   Deser, Jackiw & Templeton '82

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   asymptotically flat adapted to lightlike infinity $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}\bar{s}^2 = -\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,\mathrm{d}\varphi^2$$

$$\begin{aligned}
g_{uu} &= h_{uu} + O(\tfrac{1}{r}) \\
g_{ur} &= -1 + h_{ur}/r + O(\tfrac{1}{r^2}) \\
g_{u\varphi} &= h_{u\varphi} + O(\tfrac{1}{r}) \\
g_{rr} &= h_{rr}/r^2 + O(\tfrac{1}{r^3}) \\
g_{r\varphi} &= h_1(\varphi) + h_{r\varphi}/r + O(\tfrac{1}{r^2}) \\
g_{\varphi\varphi} &= r^2 + (h_2(\varphi) + u h_3(\varphi))r + O(1)
\end{aligned}$$

Barnich & Compere '06
Bagchi, Detournay & DG '12

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
   Obtain canonical boundary charges

$$Q_{M_n} = \frac{1}{16\pi G} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(h_{uu} + h_3\big)$$

$$\begin{aligned} Q_{L_n} = {} & \frac{1}{16\pi G\mu} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(h_{uu} + \partial_u h_{ur} + \tfrac{1}{2}\partial_u^2 h_{rr} + h_3\big) \\ & + \frac{1}{16\pi G} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(inuh_{uu} + inh_{ur} + 2h_{u\varphi} + \partial_u h_{r\varphi} \\ & \qquad - (n^2 + h_3)h_1 - inh_2 - in\partial_\varphi h_1\big) \end{aligned}$$

Bagchi, Detournay & DG '12

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

with central charges

$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

Note:

- $c_L = 0$ in Einstein gravity
- $c_M = 0$ in conformal Chern–Simons gravity ($\mu \to 0$, $\mu G = \frac{1}{8k}$)
  Flat space chiral gravity!

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Flat space Einstein and chiral gravity

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5. Improve to quantum ASA
   Trivial here

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5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
   - Straightforward in flat space chiral gravity
   - Difficult/impossible otherwise

Apply algorithm just described to flat space theories

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5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
   Monster CFT in flat space chiral gravity
   Witten '07
   Li, Song & Strominger '08
   Bagchi, Detournay & DG '12

$$Z(q) = J(q) = \frac{1}{q} + (1 + 196883)\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

Apply algorithm just described to flat space theories

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2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
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5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify
   We are happy!

Flat space chiral gravity

Bagchi, Detournay, DG '12

Conjecture:

Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \left(\Gamma \wedge \mathrm{d}\Gamma + \tfrac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) + \text{flat space bc's}$$

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- Trace and gravitational anomalies match

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Missing: full partition function on gravity side

$$Z(q) = J(q) = \frac{1}{q} + 196884\, q + \mathcal{O}(q^2)$$

Cosmic evolution from phase transition

Flat space version of Hawking–Page phase transition

Hot flat space $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}s^2 = \pm\,\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\;\mathrm{d}\varphi^2$$

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Flat space version of Hawking–Page phase transition

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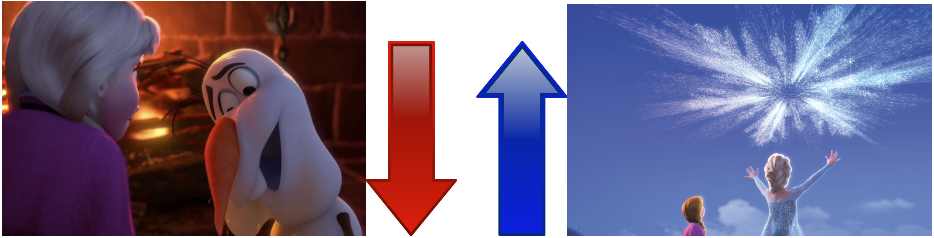

$$\mathrm{d}s^2 = \pm\,\mathrm{d}\tau^2 + \frac{(E\tau)^2\;\mathrm{d}x^2}{1+(E\tau)^2} + \big(1+(E\tau)^2\big)\Big(\mathrm{d}y + \frac{(E\tau)^2}{1+(E\tau)^2}\;\mathrm{d}x\Big)^2$$

Flat space cosmology $\qquad (y \sim y + 2\pi r_0)$

Bagchi, Detournay, DG & Simon '13

Flat space cosmologies (Cornalba & Costa '02)

- Start with BTZ in AdS:

$$\mathrm{d}s^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{R_+ r_-}{\ell r^2}\,\mathrm{d}t\right)^2$$

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- Take the $\ell \to \infty$ limit (with $R_+ = \ell\hat{r}_+$ and $r_- = r_0$)

$$\mathrm{d}s^2 = \hat{r}_+^2\left(1 - \frac{r_0^2}{r^2}\right)\,\mathrm{d}t^2 - \frac{r^2\;\mathrm{d}r^2}{\hat{r}_+^2\,(r^2 - r_0^2)} + r^2\left(\mathrm{d}\varphi - \frac{\hat{r}_+ r_0}{r^2}\;\mathrm{d}t\right)^2$$

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- Note peculiarity: no conical singularity, but asymptotic conical defect!

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Question we want to address:

Is FSC or HFS the preferred Euclidean saddle?

Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

$$Z(T,\,\Omega) = \int \mathcal{D}g\, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} \times Z_{\text{fluct.}}$$

boundary conditions specified by temperature T and angular velocity Ω

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Two Euclidean saddle points in same ensemble if

- same temperature $T = 1/\beta$ and angular velocity Ω
- obey flat space boundary conditions
- solutions without conical singularities

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Two Euclidean saddle points in same ensemble if

- same temperature $T=1/\beta$ and angular velocity Ω
- obey flat space boundary conditions
- solutions without conical singularities

Periodicities fixed:

$$(\tau_E,\,\varphi)\sim(\tau_E+\beta,\,\varphi+\beta\Omega)\sim(\tau_E,\,\varphi+2\pi)$$

Results

On-shell action:

$$\Gamma = -\frac{1}{16\pi G_N} \int \mathrm{d}^3x\sqrt{g}\,R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\text{GHY!}} \int \mathrm{d}^2x\sqrt{\gamma}\,K$$

Results

On-shell action:

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Free energy:

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

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Free energy:

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- $r_+ > 1$: FSC dominant saddle
- $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS "melts" into FSC at $T > T_c$

Entanglement entropy of Galilean CFTs and flat space holography

Bagchi, Basu, DG, Riegler '14

Using methods similar to CFT:

$$S_{\text{EE}}^{\text{GCFT}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6}\,\frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

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$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
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and

- ℓ_x: spatial distance
- ℓ_y: temporal distance
- a: UV cutoff (lattice size)

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Same results obtained holographically!

- Using methods similar to Ammon, Castro Iqbal '13, de Boer, Jottar '13, Castro, Detournay, Iqbal, Perlmutter '14
- geodesics $\Rightarrow$ Wilson lines

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13, Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\to$ spin 3 $\sim$ sl$(2) \to$ sl(3)

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- AdS gravity in CS formulation: spin 2 → spin 3 ∼ $\mathrm{sl}(2) \to \mathrm{sl}(3)$
- Flat space: similar!

$$S_{\mathrm{CS}}^{\mathrm{flat}} = \frac{k}{4\pi} \int \mathrm{CS}(\mathcal{A})$$

with $\mathrm{isl}(3)$ connection ($e^a =$ "zuvielbein")

$$\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n,\, V_m) \quad J_a = (L_n,\, U_m)$$

$\mathrm{isl}(3)$ algebra (spin 3 extension of global part of BMS/GCA algebra)

$$[L_n,\, L_m] = (n-m) L_{n+m}$$
$$[L_n,\, M_m] = (n-m) M_{n+m}$$
$$[L_n,\, U_m] = (2n-m) U_{n+m}$$
$$[M_n,\, U_m] = [L_n,\, V_m] = (2n-m) V_{n+m}$$
$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8) L_{n+m}$$
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- Same type of boundary conditions as for spin 2:

$$\mathcal{A}(r, t, \varphi) = b^{-1}(r) \big(\mathrm{d} + a(t, \varphi) + o(1) \big) b(r)$$

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- Flat space boundary conditions: $b(r) = \exp\big(\frac{1}{2}\, r M_{-1}\big)$ and

$$a(t,\, \varphi) = \big(M_1 - M(\varphi) M_{-1} - V(\varphi) V_{-2}\big)\, \mathrm{d}t + \big(L_1 - M(\varphi) L_{-1} - V(\varphi) U_{-2} - L(\varphi) M_{-1} - Z(\varphi) V_{-2}\big)\, \mathrm{d}\varphi$$

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$$+ \big(L_1 - M(\varphi)L_{-1} - V(\varphi)U_{-2} - L(\varphi)M_{-1} - Z(\varphi)V_{-2}\big)\,\mathrm{d}\varphi$$

- Spin 3 charges:

$$Q[\varepsilon_M,\, \varepsilon_L,\, \varepsilon_V,\, \varepsilon_U] \sim \oint \big(\varepsilon_M(\varphi)M(\varphi)+\varepsilon_L(\varphi)L(\varphi)+\varepsilon_V(\varphi)V(\varphi)+\varepsilon_U(\varphi)U(\varphi)\big)$$

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra

Flat space higher spin gravity

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$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} - \frac{96\big(c_L+\frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

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$$\Lambda_n = \sum_p :L_p\,M_{n-p}: \; - \tfrac{3}{10}\,(n+2)(n+3)M_n \qquad \Theta_n = \sum_p M_p\,M_{n-p}$$

other commutators as in isl(3) with $n \in \mathbb{Z}$

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- Note quantum shift and poles in central terms!
- Analysis generalizes to flat space contractions of other W-algebras

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Unitarity leads to further contraction DG, Riegler, Rosseel '14

Facts:

- Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)

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Higher spin states decouple and become null states!

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Generic flat space W-algebras DG, Riegler, Rosseel '14

1. NO–GO:

Generically (see paper) you can have only two out of three:

- Unitarity
- Flat space
- Non-trivial higher spin states

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Example:

Flat space chiral gravity

Bagchi, Detournay, DG, 1208.1658

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Example:

Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

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Example:

Flat space higher spin gravity (Galilean W_3 algebra)

Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768

Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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2. YES–GO:
There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

Unitary, non-trivial flat space higher spin algebra exists!
Vasiliev-type flat space chiral higher spin gravity?

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Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

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$$[\mathcal{V}^i_m, \mathcal{V}^j_n] = \sum_{r=0}^{\lfloor \frac{i+j}{2} \rfloor} g^{ij}_{2r}(m,n)\, \mathcal{V}^{i+j-2r}_{m+n} + c^i_{\mathcal{V}}(m)\, \delta^{ij}\, \delta_{m+n,0}$$

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where

$$c^i_{\mathcal{V}}(m) = \#(i,\, m) \times c \qquad \text{and} \qquad c = -\bar{c}$$

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- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity $\rightarrow$ Vasiliev type analogue?)

Adding chemical potentials Gary, DG, Riegler, Rosseel '14

Long story short:

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Line-element with spin-2 and spin-3 chemical potentials:

$$g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \Big(r^2\left(\mu_{\mathrm{L}}^2 - 4\mu_{\mathrm{U}}''\mu_{\mathrm{U}} + 3\mu_{\mathrm{U}}'^{\,2} + 4\mathcal{M}\mu_{\mathrm{U}}^2\right) + r\,g_{uu}^{(r)} + g_{uu}^{(0)} + g_{uu}^{(0')}\Big)\,\mathrm{d}u^2 +$$
$$\Big(r^2\mu_{\mathrm{L}} - r\mu_{\mathrm{M}}' + \mathcal{N}(1+\mu_{\mathrm{M}}) + 8\mathcal{Z}\mu_{\mathrm{V}}\Big)\,2\,\mathrm{d}u\,\mathrm{d}\varphi - (1+\mu_{\mathrm{M}})\,2\,\mathrm{d}r\,\mathrm{d}u + r^2\,\mathrm{d}\varphi^2$$

$$g_{uu}^{(0)} = \mathcal{M}(1+\mu_{\mathrm{M}})^2 + 2(1+\mu_{\mathrm{M}})(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 16\mathcal{Z}\mu_{\mathrm{U}})$$
$$+ 16\mathcal{Z}\mu_{\mathrm{L}}\mu_{\mathrm{V}} + \tfrac{4}{3}\left(\mathcal{M}^2\mu_{\mathrm{V}}^2 + 4\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}} + \mathcal{N}^2\mu_{\mathrm{U}}^2\right)$$

Spin-3 field with same chemical potentials:

$$\Phi_{\mu\nu\lambda}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\,\mathrm{d}x^\lambda = \Phi_{uuu}\,\mathrm{d}u^3 + \Phi_{ruu}\,\mathrm{d}r\,\mathrm{d}u^2 + \Phi_{uu\varphi}\,\mathrm{d}u^2\,\mathrm{d}\varphi - \left(2\mu_{\mathrm{U}}r^2 - r\mu_{\mathrm{V}}' + 2\mathcal{N}\mu_{\mathrm{V}}\right)\mathrm{d}r\,\mathrm{d}u\,\mathrm{d}\varphi$$
$$+ \mu_{\mathrm{V}}\,\mathrm{d}r^2\,\mathrm{d}u - \left(\mu_{\mathrm{U}}'r^3 - \tfrac{1}{3}r^2(\mu_{\mathrm{V}}'' - \mathcal{M}\mu_{\mathrm{V}} + 4\mathcal{N}\mu_{\mathrm{U}}) + r\mathcal{N}\mu_{\mathrm{V}}' - \mathcal{N}^2\mu_{\mathrm{V}}\right)\mathrm{d}u\,\mathrm{d}\varphi^2$$

$$\Phi_{uuu} = r^2\left[2(1+\mu_{\mathrm{M}})\mu_{\mathrm{U}}(\mathcal{M}\mu_{\mathrm{L}} - 4\mathcal{V}\mu_{\mathrm{U}}) - \tfrac{1}{3}\mu_{\mathrm{L}}^2(\mathcal{M}\mu_{\mathrm{V}} - 4\mathcal{N}\mu_{\mathrm{U}}) + 16\mu_{\mathrm{L}}\mu_{\mathrm{U}}(\mathcal{V}\mu_{\mathrm{V}} + \mathcal{Z}\mu_{\mathrm{U}}) - \tfrac{4}{3}\mathcal{M}\mu_{\mathrm{U}}^2(\mathcal{M}\mu_{\mathrm{V}}\right.$$
$$\left. + 2\mathcal{N}\mu_{\mathrm{U}})\right] + 2\mathcal{V}(1+\mu_{\mathrm{M}})^3 + \tfrac{2}{3}(1+\mu_{\mathrm{M}})^2\left(6\mathcal{Z}\mu_{\mathrm{L}} + \mathcal{M}^2\mu_{\mathrm{V}} + 2\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\right) + 16\mu_{\mathrm{L}}\mu_{\mathrm{V}}^2(\mathcal{N}\mathcal{V} - \tfrac{1}{3}\mathcal{M}\mathcal{Z})$$
$$+ \tfrac{2}{3}(1+\mu_{\mathrm{M}})\left((\mathcal{N}\mu_{\mathrm{L}} + 16\mathcal{Z}\mu_{\mathrm{U}})(2\mathcal{M}\mu_{\mathrm{V}} + \mathcal{N}\mu_{\mathrm{U}}) + 12\mathcal{M}\mathcal{V}\mu_{\mathrm{V}}^2\right) + \tfrac{64}{3}\mathcal{Z}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 12\mathcal{Z}\mu_{\mathrm{U}})$$
$$+ \mathcal{N}^2\mu_{\mathrm{L}}^2\mu_{\mathrm{V}} + 64\mathcal{V}^2\mu_{\mathrm{V}}^3 - \tfrac{8}{27}(\mathcal{M}^3\mu_{\mathrm{V}}^3 - \mathcal{N}^3\mu_{\mathrm{U}}^3) - \tfrac{4}{9}\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(4\mathcal{M}\mu_{\mathrm{V}} + 5\mathcal{N}\mu_{\mathrm{U}}) + \sum_{n=0}^{3} r^n\,\Phi_{uuu}^{(r^n)}$$

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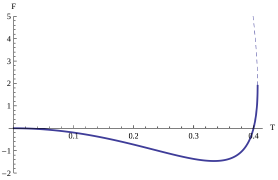
Interesting novel phase transitions of zeroth/first order:



Free energy of four branches of regular solutions as function of temperature for different values of higher spin chemical potential ratio (in AdS: see David, Ferlaino, Kumar '12)

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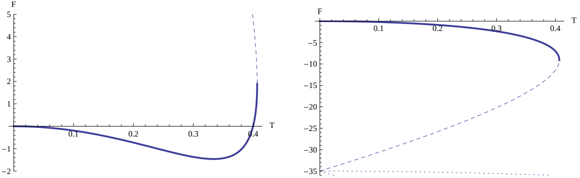

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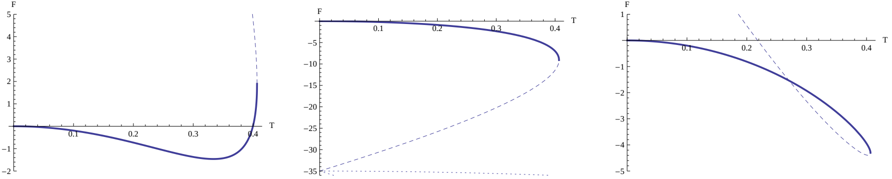

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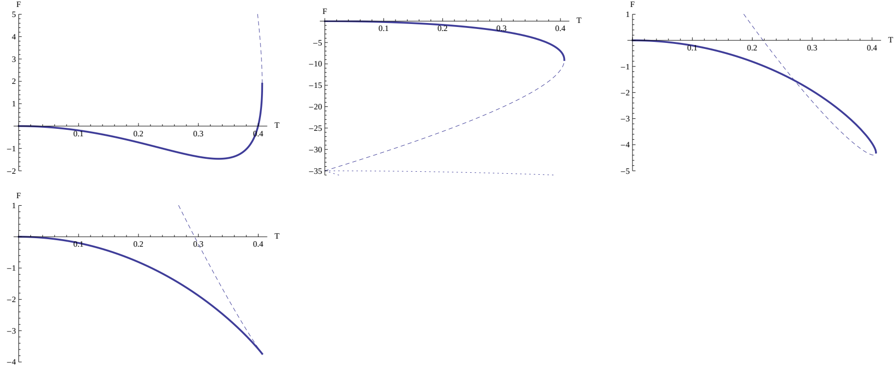

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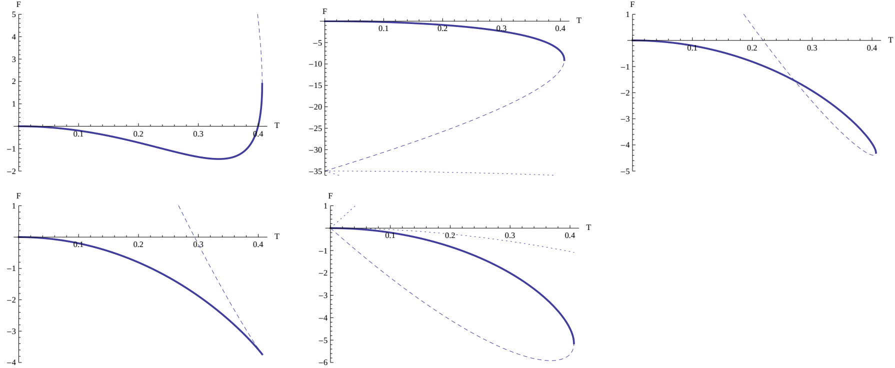

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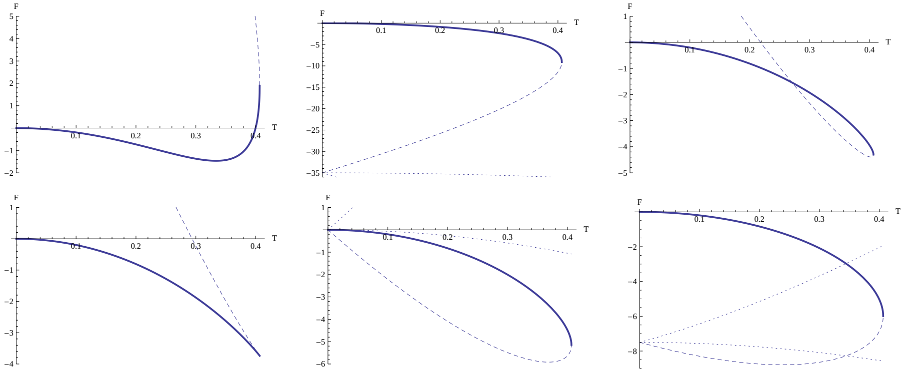

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We have answered an ϵ of the open questions.

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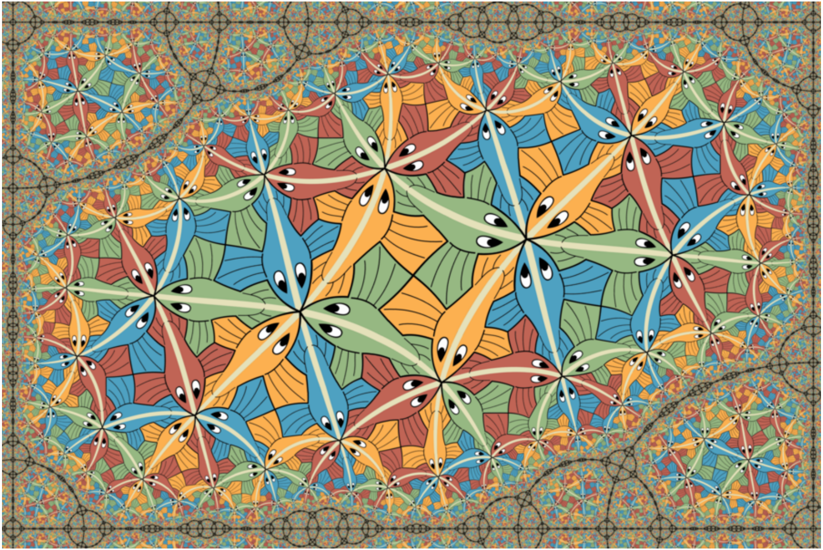
- other non-AdS holography examples?

Still missing: comprehensive family of simple models such that

- dual (conformal) field theory identified
- exists for $c \sim \mathcal{O}(1)$ (ultra-quantum limit)
- exists for $c \to \infty$ (semi-classical limit)

... or prove that no such model $\exists$, unless UV-completed to string theory!

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle