Flat Space Holography

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Brazil, September 2015

Some of our papers on flat space holography

A. Bagchi, D. Grumiller and W. Merbis,
"Stress tensor correlators in three-dimensional gravity,"
arXiv:1507.05620.

A. Bagchi, R. Basu, D. Grumiller and M. Riegler,
"Entanglement entropy in Galilean conformal field theories and flat holography,"
Phys. Rev. Lett. **114** (2015) 11, 111602 [arXiv:1410.4089].

H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel,
"Spin-3 Gravity in Three-Dimensional Flat Space,"
Phys. Rev. Lett. **111** (2013) 12, 121603 [arXiv:1307.4768].

A. Bagchi, S. Detournay, D. Grumiller and J. Simon,
"Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space,"
Phys. Rev. Lett. **111** (2013) 18, 181301 [arXiv:1305.2919].

A. Bagchi, S. Detournay and D. Grumiller,
"Flat-Space Chiral Gravity,"
Phys. Rev. Lett. **109** (2012) 151301 [arXiv:1208.1658].

Outline

Motivations

Assumptions

Flat space holography basics

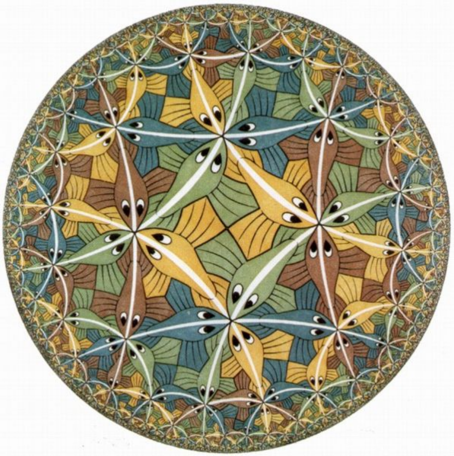
Recent results

Generalizations & open issues

Outline

Holography in our Universe?

This talk focuses on holography (in the quantum gravity sense).



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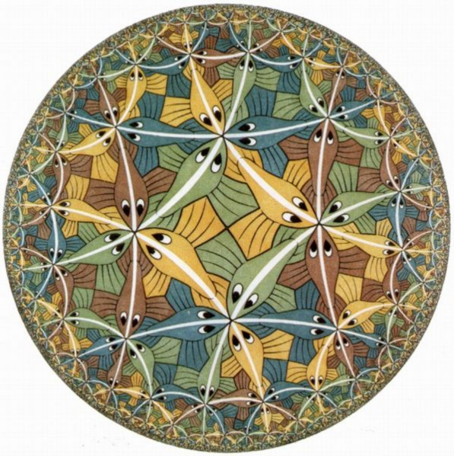
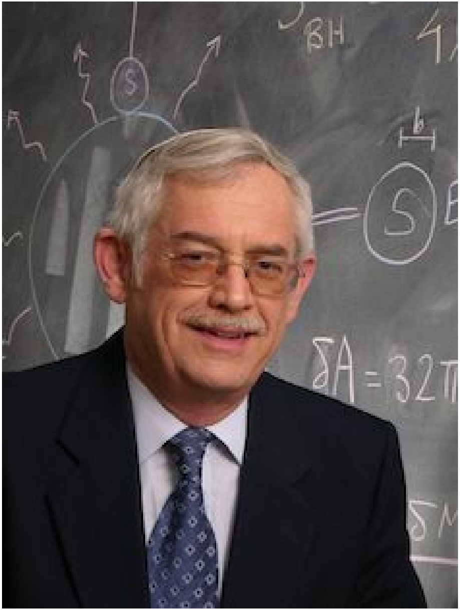


Main question: how general is holography?

In memoriam Jakob Bekenstein (May 1, 1947—August 16, 2015)



$$S = \frac{\pi A k c^3}{2 h G}$$

Testing the holographic principle

How general is holography?

- Holographic principle realized in AdS/CFT correspondence
- Special case or generic lesson for quantum gravity?

$\mathrm{AdS}_{d+1} \to \mathrm{CFT}_d$

- Use (classical) gravity to learn more about CFTs
- Strong coupling large N limit: classical gravity
- Useful tool to calculate correlation functions
- Useful tool to calculate entanglement entropy

$\mathrm{CFT}_d \to \mathrm{AdS}_{d+1}$

- Use CFTs to learn more about (quantum) gravity
- Gravity in ultra-quantum limit: simple CFT?
- Useful tool to address black hole microstates
- Useful tool for qu-gr puzzles (information paradox)

Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012

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- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson '08; Skenderis, Taylor, van Rees '09; Henneaux, Martinez, Troncoso '09; Maloney, Song, Strominger '09; DG, Sachs/Hohm '09; Gaberdiel, DG, Vassilevich '10; ... DG, Riedler, Rosseel, Zojer '13
- recent proposal by Vafa '14

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- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?

the answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., '12-'15

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- Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work at least in 2+1 dimensions
Gary, DG Rashkov '12; Afshar et al '12; Gutperle et al '14-'15; Gary, DG, Prohazka, Rey '14; ...

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- Address questions above in simple class of 3D toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries

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Address these issues in 3D!

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Interesting dichotomy:

- Either dual field theory exists $\rightarrow$ useful toy model for quantum gravity
- Or gravitational theory needs UV completion (within string theory) $\rightarrow$ indication of inevitability of string theory

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This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding

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AdS_3 gravity

- Lowest dimension with black holes and (off-shell) gravitons

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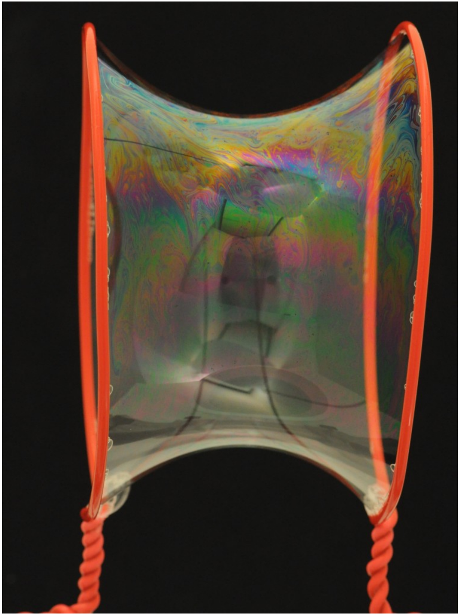
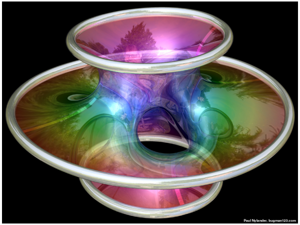
Caveat: while there are many string compactifications with AdS_3 factor, applying holography just to AdS_3 factor does not capture everything!

Picturesque analogy: soap films



Both soap films and Chern–Simons theories have

- essentially no bulk dynamics
- highly non-trivial boundary dynamics
- most of the physics determined by boundary conditions
- esthetic appeal (at least for me)



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Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

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- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0} \\
[L_n,\, M_m] &= (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0} \\
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- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
  Ashtekar, Bicak, Schmidt, '96; Barnich, Compere '06
  L_n: diffeos of circle, M_n: supertranslations, $c_{L/M}$: central extensions

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  If dual field theory exists it must be a 2D Galilean CFT!
  Bagchi et al., Barnich et al.

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- Example where it does not work: highest weight conditions

Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

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- AdS gravity in CS formulation: sl$(2)\oplus$sl(2) gauge algebra

  Achucarro, Townsend '86; Witten '88

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$$S_{\rm CS}^{\rm flat}=\frac{k}{4\pi}\int\langle\mathcal{A}\wedge\mathrm{d}\mathcal{A}+\tfrac{2}{3}\,\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\rangle$$

with isl(2) connection ($a=0,\,\pm1$)

$$\mathcal{A}=e^aM_a+\omega^aL_a$$

isl(2) algebra (global part of BMS/GCA)

$$[L_a,\,L_b]=(a-b)L_{a+b}$$
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Note: e^a dreibein, ω^a (dualized) spin-connection

Bulk EOM: gauge flatness $\to$ Einstein equations

$$\mathcal{F}=\mathrm{d}\mathcal{A}+\mathcal{A}\wedge\mathcal{A}=0$$

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- metric

$$g_{\mu\nu}\sim\tfrac{1}{2}\,\widetilde{\rm tr}\langle\mathcal{A}_\mu\mathcal{A}_\nu\rangle \quad\to\quad \mathrm{d}s^2 = M\;\mathrm{d}u^2{-}2\,\mathrm{d}u\,\mathrm{d}r{+}2N\;\mathrm{d}u\,\mathrm{d}\varphi{+}r^2\;\mathrm{d}\varphi^2$$

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Start slowly with 0-point function

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ

0-point function (on-shell action)

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- Calculate the full on-shell action Γ
- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \frac{1}{8\pi G_N}\int \mathrm{d}^2x\sqrt{\gamma}\,K - I_{\textrm{counter-term}}$$

with $I_{\textrm{counter-term}}$ chosen such that

$$\delta\Gamma\big|_{\textrm{EOM}} = 0$$

for all δg that preserve flat space bc's

0-point function (on-shell action)

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Result (Detournay, DG, Schöller, Simon '14):

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\text{GHY!}}\int \mathrm{d}^2x\sqrt{\gamma}\,K$$

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli '04
independently confirmed by Barnich, Gonzalez, Maloney, Oblak '15

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- Phase transitions?

  Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

  Evaluate Euclidean partition function in semi-classical limit

  $$Z(T,\,\Omega)=\int\mathcal{D}g\,e^{-\Gamma[g]}=\sum_{g_c}e^{-\Gamma[g_c(T,\,\Omega)]}\times Z_{\text{fluct.}}$$

  path integral bc's specified by temperature T and angular velocity Ω

  Two Euclidean saddle points in same ensemble if

  - same temperature $T=1/\beta$ and angular velocity Ω
  - obey flat space boundary conditions
  - solutions without conical singularities

  Periodicities fixed:

  $$(\tau_E,\,\varphi)\sim(\tau_E+\beta,\,\varphi+\beta\Omega)\sim(\tau_E,\,\varphi+2\pi)$$

0-point function (on-shell action)

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- Calculate the full on-shell action Γ
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  3D Euclidean Einstein gravity: for each T, Ω two saddle points:
  - Hot flat space
    $$\mathrm{d}s^2 = \mathrm{d}\tau_E^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$$
  - Flat space cosmology
    $$\mathrm{d}s^2 = r_+^2\,\big(1-\frac{r_0^2}{r^2}\big)\,\mathrm{d}\tau_E^2 + \frac{r^2\,\mathrm{d}r^2}{r_+^2\,(r^2-r_0^2)} + r^2\,\big(\mathrm{d}\varphi - \frac{r_+r_0}{r^2}\,\mathrm{d}\tau_E\big)^2$$
    shifted-boost orbifold, see Cornalba, Costa '02

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- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

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$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- Result of this comparison
  - $r_+ > 1$: FSC dominant saddle
  - $r_+ < 1$: HFS dominant saddle

  Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

  HFS "melts" into FSC at $T > T_c$

  Bagchi, Detournay, DG, Simon '13

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3:

$$\delta\Gamma\big|_{\mathrm{EOM}} \sim \int_{\partial\mathcal{M}} \mathrm{vev} \times \delta\,\mathrm{source} \sim \int_{\partial\mathcal{M}} T^{\mu\nu}_{\mathrm{BY}} \times \delta g^{\mathrm{NN}}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\mathrm{BY}}$ follows from canonical analysis as well (conserved charges)

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- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

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In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

everything works (Detournay, DG, Schöller, Simon, '14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G} \qquad N = \frac{g_{t\varphi}}{4G}$$

full tower of canonical charges: see Barnich, Compere '06

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1,\varphi_1)\, M(u_2,\varphi_2)\rangle = 0$$

$$\langle M(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$

$$\langle N(u_1,\varphi_1)\, N(u_2,\varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

Fourier modes of Galilean CFT stress tensor on cylinder:

$$M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24}$$

$$N := \sum_n \big(L_n - inuM_n\big)e^{-in\varphi} - \frac{c_L}{24}$$

Conservation equations: $\partial_u M = 0$, $\partial_u N = \partial_\varphi M$

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- Do not calculate second variation of action

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> Summarize first how this works in the AdS case

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Illustrate shortcut in $\mathrm{AdS}_3/\mathrm{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z \, \mu(z, \bar{z}) T(z)$$

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- On gravity side exploit $\mathrm{sl}(2)$ CS formulation with chemical potentials

$$A = b^{-1}(\mathrm{d}+a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar{z}} = \mu L_+ + \dots$$

Drinfeld, Sokolov '84, Polyakov '87, H. Verlinde '90
Bañados, Caro '04

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- Solve them using the Green function on the plane $G = \ln\left(z_{12}\bar{z}_{12}\right)$

$$\mathcal{L}^{(1)}(z) = -\frac{k}{2}\,\partial_{z_1}^4 G(z_{12}) = \frac{3k}{z_{12}^4}$$

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- This is the correct CFT 2-point function on the plane with $c = 6k$
- Generalize to cylinder

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Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)

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- Write EOM to first subleading order in $\epsilon_{M/L}$

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- Solve with Green function on cylinder

$$M^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \qquad N^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L\,\tau_{12})}{s_{12}^4}$$

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- Correct 2-point functions for Einstein gravity with $c_L = 0$, $c_M = 12k$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

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- Iteratively solve EOM

$$\partial_u M=-k\partial_\varphi^3\mu_L+\mu_L\partial_\varphi M+2M\partial_\varphi\mu_L$$
$$\partial_u N=-k\partial_\varphi^3\mu_M+(1+\mu_M)\partial_\varphi M+2M\partial_\varphi\mu_M+\mu_L\partial_\varphi N+2N\partial_\varphi\mu_L$$

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- Result on gravity side matches precisely Galilean CFT results

$$\langle M^1\,N^2\,N^3\rangle = \frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2} \qquad \langle N^1\,N^2\,N^3\rangle = \frac{c_L - c_M\,\tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2}$$

provided we choose again the Einstein values $c_L = 0$ and $c_M = 12k$

4-point functions (enter cross-ratios)

First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points

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- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1\, N^2\, N^3\, N^4\rangle = \frac{2c_M\, g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

$$\langle N^1\, N^2\, N^3\, N^4\rangle = \frac{2c_L\, g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

with the cross-ratio function

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = \frac{s_{12}\, s_{34}}{s_{13}\, s_{24}}$$

and

$$\Delta_4 = 4g_4'(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)$$

$$\eta_{1234} = \sum(-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)$$

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First correlators with non-universal function of cross-ratios

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and

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$$\eta_{1234} = \sum(-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)$$

- Gravity side yields precisely the same result!

5-point functions (further check of consistency of flat space holography)

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with the previous definitions and ($\zeta = \frac{s_{25}\, s_{34}}{s_{35}\, s_{24}}$)

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Shortcut to 42 (Bagchi, DG, Merbis '15)

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- We can also derive same recursion relations on gravity side!

n-point functions in flat space holography
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Fairly non-trivial check that 3D flat space holography can work!

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Some further checks that dual field theory is Galilean CFT:

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$$S_{\text{EE}}^{\text{GCFT}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6}\,\frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

  Calculation on gravity side confirms result above
  (using Wilson lines in CS formulation)

Outline

Generalizations & open issues

Recent generalizations:

- adding chemical potentials

  Works! (Gary, DG, Riegler, Rosseel '14)

  In CS formulation:

$$A_0 \to A_0 + \mu$$

Recent generalizations:

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- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)

Conformal CS gravity at level $k = 1$ with flat space boundary conditions conjectured to be dual to chiral half of monster CFT.
Action (gravity side):

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \mathrm{d}^3x \sqrt{-g}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Partition function (field theory side, see Witten '07):

$$Z(q) = J(q) = \frac{1}{q} + 196884\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi +$quantum corrections

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- generalization to supergravity

  Works! (Barnich, Donnay, Matulich, Troncoso '14)

  Asymptotic symmetry algebra = super-BMS$_3$

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  Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13)

  New type of algebra: W-like BMS ("BMW")

$$
\begin{aligned}
[U_n,\, U_m] &= (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} \\
&\quad - \frac{96\big(c_L + \frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0} \\
[U_n,\, V_m] &= (n-m)(2n^2+2m^2-nm-8)M_{n+m} + \frac{96}{c_M}(n-m)\Theta_{n+m} \\
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\end{aligned}
$$

$[L,L]$, $[L,M]$,$[M,M]$ as in BMS_3 $\quad$ $[L,U]$, $[L,V]$, $[M,U]$, $[M,V]$ as in isl(3)

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- Further checks in 3D (n-point correlators, partition function, ...)

  Barnich, Gonzalez, Maloney, Oblak '15: 1-loop partition function matches BMS_3 character

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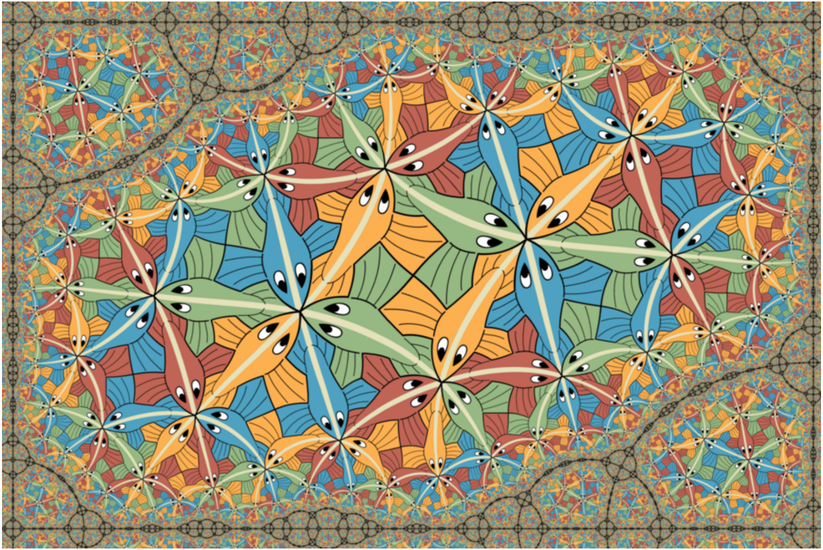
> - holography more general than AdS/CFT
> - (when) does it work even more generally?

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle