Flat space higher spin holography

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COST Workshop "The String Theory Universe" Mainz, September 2014



based on work with Afshar, Bagchi, Detournay, Fareghbal, Gary, Rey, Riegler, Rosseel, Salzer, Schöller, Simon, ...

Outline

Motivation (how general is holography?)

Simplification (3D)

Generalization (higher derivatives or spins)

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Motivation How general is holography?

> Holographic principle, if correct, must work beyond AdS/CFT holographic principle: 't Hooft '93; Susskind '94

AdS/CFT precursor: Brown, Henneaux '86 AdS/CFT: Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98

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non-unitary holography:
AdS/log CFT '08-'13: review: DG, Riedler, Rosseel, Zojer '13
Vafa '14
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- Holographic principle, if correct, must work beyond AdS/CFT
- Does it work in flat space?
 Polchinski '99
 Susskind '99
 Giddings '00
 Gary, Giddings, Penedones '09; Gary, Giddings '09; ...

How general is holography?

- Holographic principle, if correct, must work beyond AdS/CFT
- Does it work in flat space?
- Can we find models realizing flat space/field theory correspondences? Barnich, Compere '06 Barnich et al. '10-'14 Bagchi et al. '10-'14 Strominger et al. '13-'14

...

flat space chiral gravity: Bagchi, Detournay, DG '12

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- Can we find models realizing flat space/field theory correspondences?
- Are there higher-spin versions of such models? Afshar, Bagchi, Fareghbal, DG, Rosseel '13 Gonzalez, Matulich, Pino, Troncoso '13

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part of larger program: non-AdS holography in higher spin gravity
Gary, DG, Rashkov '12
Afshar, Gary, DG, Rashkov, Riegler '12
Gutperle, Hijano, Samani '13
Gary, DG, Prohazka, Rey '14
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- Relation between S-matrix and holographic observables? Strominger et al '13, '14

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Many interesting open issues in flat space holography!

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Address these issues in 3D!



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"Gravity 3D is a spellbinding experience"



... so let us consider 3D gravity!

Daniel Grumiller - Flat space higher spin holography

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Flat space analogues of these features? Does naive $\Lambda \to 0$ limit work?

Global AdS metric ($\varphi \sim \varphi + 2\pi$): $ds_{AdS}^2 = d(\ell\rho)^2 - \cosh^2(\frac{\ell\rho}{\ell}) dt^2 + \ell^2 \sinh^2(\frac{\ell\rho}{\ell}) d\varphi^2$

Global AdS metric
$$(\varphi \sim \varphi + 2\pi)$$
:
 $ds^2_{AdS} = d(\ell\rho)^2 - \cosh^2(\frac{\ell\rho}{\ell}) dt^2 + \ell^2 \sinh^2(\frac{\ell\rho}{\ell}) d\varphi^2$
Limit $\ell \to \infty \ (r = \ell\rho)$:
 $ds^2_{Flat} = dr^2 - dt^2 + r^2 d\varphi^2 = -du^2 - 2 du dr + r^2 d\varphi^2$

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BTZ metric:

$$\mathrm{d}s_{\rm BTZ}^2 = -\frac{\left(\frac{r^2}{\ell^2} - \frac{r^2_+}{\ell^2}\right)(r^2 - r^2_-)}{r^2} \,\mathrm{d}t^2 + \frac{r^2 \,\mathrm{d}r^2}{\left(\frac{r^2}{\ell^2} - \frac{r^2_+}{\ell^2}\right)(r^2 - r^2_-)} + r^2 \left(\mathrm{d}\varphi - \frac{\frac{r_+}{\ell} r_-}{r^2} \,\mathrm{d}t\right)^2$$

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$$\begin{split} \mathrm{d}s_{\rm BTZ}^2 &= -\frac{(\frac{r^2}{\ell^2} - \frac{r_+^2}{\ell^2})(r^2 - r_-^2)}{r^2} \,\,\mathrm{d}t^2 + \frac{r^2 \,\,\mathrm{d}r^2}{(\frac{r^2}{\ell^2} - \frac{r_+^2}{\ell^2})(r^2 - r_-^2)} + r^2 \left(\,\mathrm{d}\varphi - \frac{\frac{r_+}{\ell} \,\,r_-}{r^2} \,\,\mathrm{d}t\right)^2\\ \text{Limit } \ell \to \infty \,\left(\hat{r}_+ = \frac{r_+}{\ell} = \text{finite}\right):\\ \mathrm{d}s_{\rm FSC}^2 &= \hat{r}_+^2 \left(1 - \frac{r_-^2}{r^2}\right) \,\,\mathrm{d}t^2 - \frac{1}{1 - \frac{r_-^2}{r^2}} \frac{\mathrm{d}r^2}{\hat{r}_+^2} + r^2 \left(\,\mathrm{d}\varphi - \frac{\hat{r}_+ \,\,r_-}{r^2} \,\,\mathrm{d}t\right)^2 \end{split}$$

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$$ds_{\rm BTZ}^{2} = -\frac{\left(\frac{r^{2}}{\ell^{2}} - \frac{r^{2}_{+}}{\ell^{2}}\right)(r^{2} - r^{2}_{-})}{r^{2}} dt^{2} + \frac{r^{2} dr^{2}}{\left(\frac{r^{2}}{\ell^{2}} - \frac{r^{2}_{+}}{\ell^{2}}\right)(r^{2} - r^{2}_{-})} + r^{2} \left(d\varphi - \frac{\frac{r_{+}}{\ell} r_{-}}{r^{2}} dt\right)^{2}$$

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 $ds_{\rm FSC}^{2} = \hat{r}_{+}^{2} \left(1 - \frac{r^{2}_{-}}{r^{2}}\right) dt^{2} - \frac{1}{1 - \frac{r^{2}_{-}}{r^{2}}} \frac{dr^{2}}{\hat{r}_{+}^{2}} + r^{2} \left(d\varphi - \frac{\hat{r}_{+} r_{-}}{r^{2}} dt\right)^{2}$

Shifted-boost orbifold studied by Cornalba & Costa more than decade ago Describes expanding (contracting) Universe in flat space (horizon $r = r_{-}$)

FSC Penrose diagram (2D slice)



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(graphics from Bagchi, DG, Salzer, Sarkar, Schöller '14)

- I: expanding cosmology relevant patch for thermodynamics
- II: contracting cosmology
- III, IV: regions with access to singularity (wiggly line)
- i^+ : future time-like infinity
- \mathcal{J}^+ : future null infinity
- *H*⁺: horizon of expanding cosmology (dashed line)
- *H*⁻: horizon of contracting cosmology (dashed line)
- \mathcal{J}^- : past null infinity
- ▶ i[−]: past time-like infinity

Simplification (3D)
Flat space limit Example 2: Limit of asymptotic boundaries (ultra-relativistic boost)



Example 3: Limit of asymptotic symmetries (Barnich, Compère '06)

▶ Take two copies of Virasoro, generators \mathcal{L}_n , $\overline{\mathcal{L}}_n$, central charges c, \overline{c}

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$$L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n := \frac{1}{\ell} \left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n} \right)$$

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• Make ultrarelativistic boost, $\ell \to \infty$ (İnönü–Wigner contraction)

$$[L_n, L_m] = (n-m) L_{n+m} + c_L \frac{1}{12} (n^3 - n) \delta_{n+m,0}$$
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Central charges:

$$c_L = c - \bar{c}$$
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► BMS₃ = GCA₂ = URCA₂ Bagchi, Gopakumar '09, Bagchi '10

CS with weird boundary conditions (Achucarro & Townsend '86; Witten '88; Bañados '96)

► CS action:

$$S_{\rm CS} = \frac{k}{4\pi} \int \mathrm{CS}(A) - \frac{k}{4\pi} \int \mathrm{CS}(\bar{A})$$

with

$$\operatorname{CS}(A) = \langle A \wedge \mathrm{d}A + \frac{2}{3}A \wedge A \wedge A \rangle$$

Locally trivial (pure gauge), but globally non-trivial

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Virasoro charges and algebra:

$$Q[\varepsilon] \sim \oint \varepsilon(x^+) \mathcal{L}(x^+) \qquad \delta_{\varepsilon} \mathcal{L} = \mathcal{L}' \varepsilon + 2\mathcal{L} \varepsilon' + \frac{c}{12} \varepsilon'''$$

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► BMS charges and BMS/GCA algebra:

$$Q[\varepsilon_M, \varepsilon_L] \sim \oint (\varepsilon_M(\varphi)M(\varphi) + \varepsilon_L(\varphi)L(\varphi))$$

 $\delta_{\varepsilon_L} L = L' \, \varepsilon_L + 2L \, \varepsilon_L' + \frac{c_L}{12} \, \varepsilon_L''' \quad \delta_{\varepsilon_L} M = M' \, \varepsilon_L + 2M \, \varepsilon_L' + \frac{c_M}{12} \, \varepsilon_L''' \quad \delta_{\varepsilon_M} M = 0$

Graphics by Barnich, Gomberoff, Gonzalez '12

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- Geometric meaning:

$$ds^{2} = \mathcal{M}(\varphi) \, du^{2} - 2 \, du \, dr + r^{2} \, d\varphi^{2} + 2\mathcal{L}(\varphi) \, du \, d\varphi$$

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- Flat space cosmologies: $\mathcal{M} = M \ge 0$, $\mathcal{L} = J$



 Direct calculation of Cardy-like formula in GCA₂ with c_L = 0 (Bagchi, Detournay, Simón '12, Barnich '12)

$$S_{\rm GCA} = \pi h_L \sqrt{\frac{c_M}{6h_M}}$$

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Note the unusual sign between the two terms!

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Get contracted Cardy-like formula:

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4

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 Einstein gravity: c_L = 0 reproduces correct formula for S_{GCA} of Bagchi, Detournay, Simón '12, Barnich '12 Phase transitions Statement of main result

Hot flat space

$$(\varphi \sim \varphi + 2\pi)$$

$$\mathrm{d}s^2 = \pm \,\mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \,\mathrm{d}\varphi^2$$

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$$ds^{2} = \pm d\tau^{2} + \frac{(E\tau)^{2} dx^{2}}{1 + (E\tau)^{2}} + \left(1 + (E\tau)^{2}\right) \left(dy + \frac{(E\tau)^{2}}{1 + (E\tau)^{2}} dx\right)^{2}$$

Flat space cosmology Bagchi, Detournay, Grumiller, Simón '13

Daniel Grumiller - Flat space higher spin holography

Simplification (3D)

 $(y \sim y + 2\pi r_0)$

> Evaluate Euclidean partition function in semi-classical limit

$$Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}}$$

boundary conditions specify temperature T , angular velocity $\boldsymbol{\Omega}$

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For this to work need full action $\Gamma!$

Towards a holographic dictionary 0-point function (Detournay, DG, Schöller, Simón '14)

Full (holographically renormalized) action:

$$\Gamma = -\frac{1}{16\pi G} \int_M \mathrm{d}^3 x \sqrt{g} \, R - \frac{1}{16\pi G} \int_{\partial M} \mathrm{d}^2 x \sqrt{\gamma} \, K$$

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• Phase transition at self-dual point $(r_+ = 1)$:

 $2\pi T_c = \Omega$ $T > T_c$: FSC stable $T < T_c$: HFS stable

• Sources $\psi_{\mu\nu}$: non-normalizable solutions to linearized EOM

Sources and 1-point functions (Detournay, DG, Schöller, Simón '14)

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$$\psi_{\tau\varphi} = r^2 \,\partial_\tau \xi^0 + \text{norm.} \qquad \psi_{\varphi\varphi} = 2r^2 \,\partial_\varphi \xi^0 + \text{norm.} \qquad \psi_{\tau\tau} = 0 + \text{norm.}$$

with

$$\xi^0 = \xi_J(\varphi) \tau + \frac{1}{2} \int \mathrm{d}\varphi \,\xi_M(\varphi)$$

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Result for 1-point functions then follow from usual holographic dictionary:

$$\delta \Gamma \big|_{\text{EOM}} = \frac{1}{2} \, \int_{\partial M} \mathrm{d}^2 x \left(\frac{M}{2\pi} \, \delta \xi_M - \frac{J}{2\pi} \, \delta \xi_J \right)$$

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 \blacktriangleright M and J coincide precisely with zero-point canonical charges!

Outline

Motivation (how general is holography?)

Simplification (3D)

Generalization (higher derivatives or spins)

Conformal Chern–Simons gravity at level $k=1\simeq$ chiral extremal CFT with central charge c=24

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \left(\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma \right) + \text{flat space bc's}$$

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Missing: full partition function on gravity side

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$

Afshar, Bagchi, Fareghbal, DG, Rosseel '13, Gonzalez, Matulich, Pino, Troncoso '13

▶ AdS gravity in CS formulation: spin 2 → spin 3 \sim sl(2) → sl(3)

Afshar, Bagchi, Fareghbal, DG, Rosseel '13, Gonzalez, Matulich, Pino, Troncoso '13

AdS gravity in CS formulation: spin 2 → spin 3 ~ sl(2) → sl(3)
 Flat space: similar!

$$S_{\rm CS}^{\rm flat} = \frac{k}{4\pi} \int {\rm CS}(\mathcal{A})$$

with $\operatorname{isl}(3)$ connection ($e^a=\text{``zuvielbein''}$)

$$\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n, V_m) \quad J_a = (L_n, U_m)$$

 $\mathsf{isl}(3)$ algebra (spin 3 extension of global part of $\mathsf{BMS}/\mathsf{GCA}$ algebra)

$$[L_n, L_m] = (n - m)L_{n+m}$$

$$[L_n, M_m] = (n - m)M_{n+m}$$

$$[L_n, U_m] = (2n - m)U_{n+m}$$

$$[M_n, U_m] = [L_n, V_m] = (2n - m)V_{n+m}$$

$$[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m}$$

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Same type of boundary conditions as for spin 2:

$$\mathcal{A}(r, t, \varphi) = b^{-1}(r) \left(d + a(t, \varphi) + o(1) \right) b(r)$$

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Spin 3 charges:

$$Q[\varepsilon_M, \varepsilon_L, \varepsilon_V, \varepsilon_U] \sim \oint \left(\varepsilon_M(\varphi) M(\varphi) + \varepsilon_L(\varphi) L(\varphi) + \varepsilon_V(\varphi) V(\varphi) + \varepsilon_U(\varphi) U(\varphi)\right)$$

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

▶ Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W₃-algebra

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- ▶ Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W₃-algebra
- Obtain new type of W-algebra as extension of BMS ("BMW")

$$\begin{split} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c_L}{12} \left(n^3 - n\right) \delta_{n+m, 0} \\ [L_n, M_m] &= (n-m)M_{n+m} + \frac{c_M}{12} \left(n^3 - n\right) \delta_{n+m, 0} \\ [U_n, U_m] &= (n-m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M} (n-m)\Lambda_{n+m} \\ &- \frac{96 \left(c_L + \frac{44}{5}\right)}{c_M^2} (n-m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0} \\ [U_n, V_m] &= (n-m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M} (n-m)\Theta_{n+m} \\ &+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0} \\ \Lambda_n &= \sum_p : L_p M_{n-p} : -\frac{3}{10} (n+2)(n+3)M_n \qquad \Theta_n = \sum_p M_p M_{n-p} \end{split}$$

other commutators as in isl(3) with $n \in \mathbb{Z}$

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$$\begin{split} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c_L}{12} \left(n^3 - n\right) \delta_{n+m,0} \\ [L_n, M_m] &= (n-m)M_{n+m} + \frac{c_M}{12} \left(n^3 - n\right) \delta_{n+m,0} \\ [U_n, U_m] &= (n-m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M} (n-m)\Lambda_{n+m} \\ &- \frac{96 \left(c_L + \frac{44}{5}\right)}{c_M^2} (n-m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m,0} \\ [U_n, V_m] &= (n-m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M} (n-m)\Theta_{n+m} \\ &+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m,0} \end{split}$$

Note quantum shift and poles in central terms!

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

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- \blacktriangleright Analysis generalizes to flat space contractions of other $W\mbox{-algebras}$

Unitarity in flat space Unitarity leads to further contraction DG, Riegler, Rosseel '14

Facts:

• Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)

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Higher spin states decouple and become null states!
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Generically (see paper) you can have only two out of three:

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Example:

Flat space chiral gravity Bagchi, Detournay, DG, 1208.1658

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Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

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Flat space higher spin gravity (Galilean W_3 algebra) Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768 Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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2. YES-GO:

There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

Unitary, non-trivial flat space higher spin algebra exists! Vasiliev-type flat space chiral higher spin gravity?

Flat space W_∞ -algebra compatible with unitarity DG, Riegler, Rosseel '14

► We do not know if flat space chiral higher spin gravity exists...

Flat space $\mathit{W}_\infty\textsc{-algebra}$ compatible with unitarity DG, Riegler, Rosseel '14

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- \blacktriangleright Vacuum descendants $\mathcal{W}_m^i | 0 \rangle$ are null states for all i and m!
- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity → Vasiliev type analogue?)

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- holographic entanglement entropy in flat space (higher spin) gravity
 - Long way to go before fully understanding flat space holography
 - Part of the path now seems clear and may lead to new insights
 - Other parts probably will require novel techniques

Selected references

Thanks for your attention!

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Thanks to Bob McNees for providing the LATEX beamerclass!