Soft Heisenberg Hair

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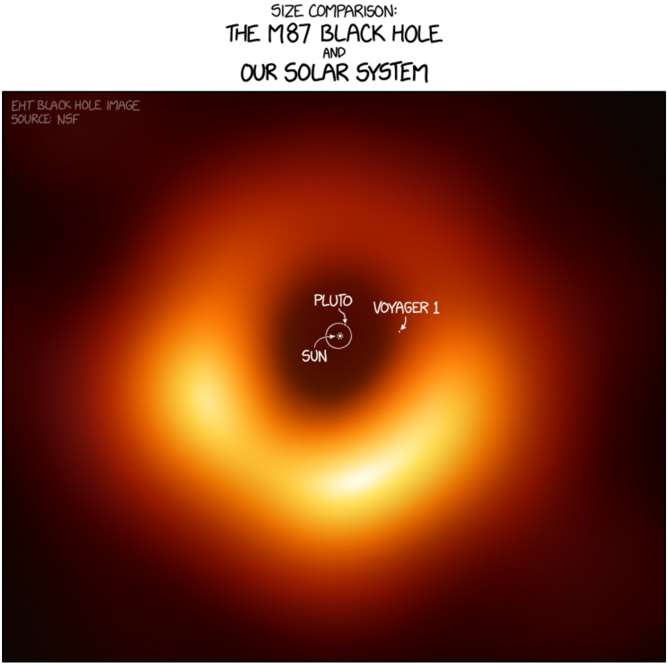
SIZE COMPARISON:
THE M87 BLACK HOLE
AND
OUR SOLAR SYSTEM
EHT BLACK HOLE IMAGE
SOURCE: NSF
PLUTO
VOYAGER 1
SUN

xkcd 2135

Two simple punchlines

1. Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
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at least in three spacetime dimensions

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based on work (2016-2019) with

- Hamid Afshar, Shahin Sheikh-Jabbari, Zahra Mirzaiyan [IPM Teheran]
- Martin Ammon [U. Jena]
- Stephane Detournay, Wout Merbis, Stefan Prohazka, Max Riegler [ULB]
- Hernán González [AIU Santiago]
- Philip Hacker, Raphaela Wutte, Céline Zwikel [TU Wien]
- Alfredo Perez, David Tempo, Ricardo Troncoso [CECs Valdivia]
- Hossein Yavartanoo [ITP Beijing]

Outline

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Physics with boundaries

Science is a differential equation. Religion is a boundary condition. — Alan Turing

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- Gauge or gravity theories in presence of (asymptotic) boundaries: asymptotic symmetries

Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt's) modulo trivial gauge transformations

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- Choice of boundary conditions determines asymptotic symmetries

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Asymptotic symmetries in gravity

- Impose some bc's at (asymptotic or actual) boundary:

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- typically, Killing vectors can be expanded radially

$$\xi^\mu(r_b,\,x^i) = \xi^\mu_{(0)}(r_b,\,x^i) + \text{subleading terms}$$

$\xi^\mu_{(0)}(r_b,\,x^i)$: generates asymptotic symmetries
subleading terms: generate trivial diffeos

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Definition of asymptotic symmetry algebra

Lie bracket quotient algebra of asymptotic Killing vectors modulo trivial diffeos

Simple example (based on unpublished notes with Salzer)

Asymptotic Rindler$_2$ spacetimes (in Eddington–Finkelstein gauge)

- Consider class of 2d metrics, partially gauge-fixed

$$g_{rr}(r,\,u) = 0$$
$$g_{ur}(r,\,u) = -1$$
$$g_{uu}(r,\,u) = \delta g(u) r + \mathcal{O}(1)$$

expanded for large r

Note: Ricci scalar tends to zero for large r

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- asymptotic symmetry algebra ("BMS$_2$"):

$$\big[\xi(\epsilon_1,\,\eta_1),\,\xi(\epsilon_2,\,\eta_2)\big]_{\text{Lie}} = \xi\big(\epsilon_1\epsilon_2' - \epsilon_2\epsilon_1',\,(\epsilon_1\eta_2 - \epsilon_2\eta_1)'\big)$$

  Lie bracket algebra of asymptotic Killing vectors is infinite dimensional here

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- in Fourier-modes $L_n := \xi(\epsilon = ie^{inu},\,\eta = 0)$, $J_n := \xi(\epsilon = 0,\,\eta = ie^{inu})$:

$$[L_n,\,L_m]_{\text{Lie}} = (n-m)\,L_{n+m} \qquad [J_n,\,J_m]_{\text{Lie}} = 0 \qquad [L_n,\,J_m]_{\text{Lie}} = -(n+m)\,J_{n+m}$$

Witt algebra (spin-2) with current-type algebra (spin-0)

Simple example (based on unpublished notes with Salzer)

Asymptotic Rindler$_2$ spacetimes (asymptotically in Eddington–Finkelstein gauge)

- Consider class of 2d metrics

$$g_{rr}(r,\,u)=0+\mathcal{O}(1/r)$$
$$g_{ur}(r,\,u)=-1+\mathcal{O}(1/r)$$
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- dropping partial gauge-fixing does not change asymptotic symmetries instead, switches on trivial diffeos

Canonical boundary charges

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  time-independent Schrödinger equation:

  $$-\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x)$$

  look for (normalizable) bound state solutions, $E < 0$
  - Dirichlet bc's: no bound states
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  - Robin bc's

    $$\big(\psi + \alpha\psi'\big)\big|_{x=0^+} = 0 \qquad \alpha \in \mathbb{R}^+$$

    lead to one bound state

    $$\psi(x)\big|_{x\geq 0} = \sqrt{\frac{2}{\alpha}}\, e^{-x/\alpha}$$

    with energy $E = -1/\alpha^2$, localized exponentially near $x = 0$

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: either use Noether's second theorem and covariant phase space analysis or perform Hamiltonian analysis in presence of boundaries

  Some references:
  - covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
  - review: see Compère, Fiorucci '18 and refs. therein
  - canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
  - review: see Bañados, Reyes '16 and refs. therein

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- in Hamiltonian language: gauge generator $G[\epsilon]$ varies as

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Trivial gauge transformations generated by some ϵ with $Q[\epsilon] = 0$

Canonical realization of asymptotic symmetries

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- on constraint surface $\Gamma[\epsilon] = Q[\epsilon]$, hence

$$\delta_{\epsilon_1} Q[\epsilon_2] = \{Q[\epsilon_1],\, Q[\epsilon_2]\} = Q[\epsilon_1 \circ \epsilon_2] + Z[\epsilon_1,\, \epsilon_2]$$

Z: possible central extension of asymptotic symmetry algebra

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Poisson (or Dirac) bracket algebra of canonical boundary charges

- physical phase space falls into representations of asymptotic symmetry algebra $\Rightarrow$ useful e.g. for holography

Simple example: abelian Chern–Simons

- abelian Chern–Simons action (on cylinder)

$$I[A] = \frac{k}{4\pi} \int_{\mathbb{R}\times\Sigma} A \wedge \mathrm{d}A$$

Note: topological QFT with no local physical degrees of freedom

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$$Q[\epsilon] = \frac{k}{2\pi} \oint_{\partial\Sigma} \epsilon\, A$$

- choice of bc's

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preserved by $\epsilon = \eta(\varphi) +$ subleading

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- Fourier modes $J_n \sim \oint \mathcal{J} e^{in\varphi}$ yield $u(1)_k$ current algebra, $i\{J_n,\, J_m\} = \frac{k}{2}\, n\, \delta_{n+m,\,0}$

Edge states

see e.g. Halperin '82, Witten '89, or Balachandran, Chandar, Momen '94

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  - descendants of vacuum are examples of edge states

$$|\text{edge}(\{n_i\})\rangle = \prod_{\{n_i>0\}} J_{-n_i}|0\rangle$$

e.g.

$$|\text{edge}(\{1,1,42\})\rangle = J_{-1}^2 J_{-42}|0\rangle$$

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- theories with no local physical degrees of freedom can have edge states! $\Rightarrow$ perhaps cleanest example of holography

Outline

Motivation for near horizon boundary conditions

Old idea by Strominger '97 and Carlip '98

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

Motivations:

Motivation for near horizon boundary conditions

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Main idea

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Motivations:

- Want to ask conditional questions "given a black hole, what are the probabilities for some scattering process"
- Want to understand Bekenstein–Hawking entropy

$$S_{\text{BH}} = \frac{A}{4G} + \mathcal{O}(\ln(A/G))$$

Motivation for near horizon boundary conditions

Old idea by Strominger '97 and Carlip '98

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

Motivations:

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Explicit form of near horizon boundary conditions

See Donnay, Giribet, Gonzalez, Pino '15 and Afshar et al '16

Postulates of near horizon boundary conditions:

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Postulates of near horizon boundary conditions:

1. Rindler approximation

$$\mathrm{d}s^2 = -\kappa^2 r^2 \,\mathrm{d}t^2 + \mathrm{d}r^2 + \Omega_{ab}(t,\,x^c)\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

   $r \to 0$: Rindler horizon

   κ: surface gravity

   Ω_{ab}: metric transversal to horizon

   $\dots$: terms of higher order in r or rotation terms

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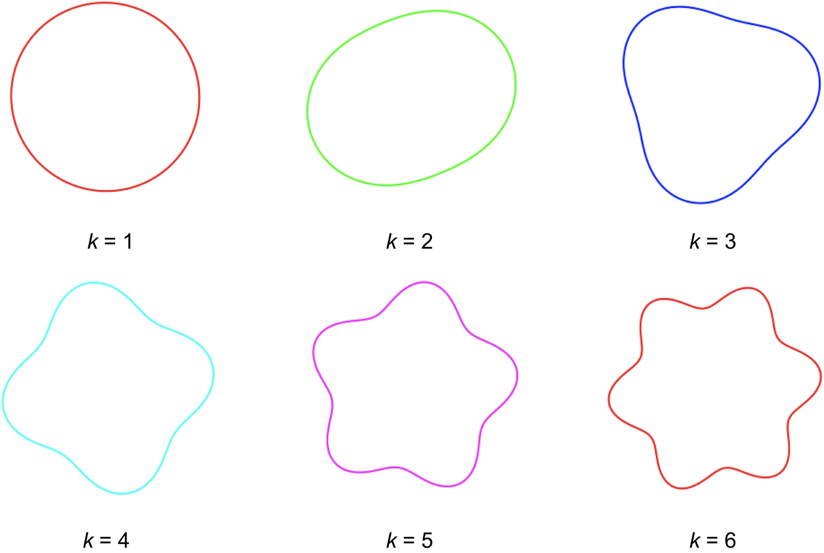
4. Remaining terms fixed by consistency of canonical boundary charges

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations



Near horizon symmetries = "asymptotic symmetries" for near horizon bc's

Restrict for the time being to AdS_3 black holes (BTZ)

Simplification in 3d:

$$\mathrm{d}s^2 = \Big[-\kappa^2 r^2\,\mathrm{d}t^2 + \mathrm{d}r^2 + \gamma^2(\varphi)\,\mathrm{d}\varphi^2 + 2\kappa\,\omega(\varphi)\,r^2\,\mathrm{d}t\,\mathrm{d}\varphi \Big]\big(1 + \mathcal{O}(r^2)\big)$$

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- Trivial or non-trivial?
  Answer provided by boundary charges!

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- Non-trivial diffeo!
- Canonical analysis yields

$$Q^{\pm}[\epsilon^{\pm}] \sim \oint \mathrm{d}\varphi\,\epsilon^{\pm}(\varphi)\,\big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

  where $\epsilon^{\pm}$ are functions appearing in asymptotic Killing vectors

  charge conservation follows from on-shell relations $\partial_t\gamma = 0 = \partial_t\omega$

  explains last word in title: γ and ω are hair of black hole

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- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm, \, \mathcal{J}_m^\pm] = \tfrac{1}{2} \, n \, \delta_{n+m,\, 0}$$

Two $u(1)$ current algebras! Afshar et al. 16

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$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = \tfrac{1}{2}\, n\, \delta_{n+m,\,0}$$

- Isomorphic to Heisenberg algebras plus center

$$[X_n,\, P_m] = i\, \delta_{n,m} \qquad\qquad [P_0,\, X_n] = 0 = [X_0,\, P_n]$$

$P_0 = \mathcal{J}_0^+ + \mathcal{J}_0^-$, $X_n = \mathcal{J}_n^+ - \mathcal{J}_{-n}^-$, $P_n = 2i/n(\mathcal{J}_{-n}^+ + \mathcal{J}_n^-)$ for $n \neq 0$

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature

   By contrast: asymptotically AdS or flat space bc's allow for black hole states at different masses and hence different temperatures

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)

   By contrast: for given temperature not all states in theories with asymptotically AdS or flat space bc's are free from conical singularities; usually a unique black hole state is picked

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)
3. There is a non-trivial reducibility parameter (= Killing vector)

   By contrast: for any other known (non-trivial) bc's there is no vector field that is Killing for all geometries allowed by bc's

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4. Technical feature: in Chern–Simons formulation of 3d gravity simple expressions in diagonal gauge

$$A^{\pm} = b^{\mp 1}\big(\,\mathrm{d}+a^{\pm}\big)\,b^{\pm 1}$$
$$a^{\pm} = L_0\Big(\big(\gamma(\varphi) \pm \omega(\varphi)\big)\,\mathrm{d}\varphi + \kappa\,\mathrm{d}t\Big)$$
$$b = \exp\big[\big(L_+ - L_-\big)\,r/2\big]$$

$L_{\pm}$ are $sl(2,\,\mathbb{R})$ raising/lowering generators
L_0 is $sl(2,\,\mathbb{R})$ Cartan subalgebra generator

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5. Leads to soft Heisenberg hair (see next slides!)

Outline

Soft Heisenberg hair for BTZ

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

$$|\text{black flower}\rangle \sim \prod_{n_i^\pm > 0} \mathcal{J}^+_{-n_i^+} \mathcal{J}^-_{-n_i^-} |\text{black hole}\rangle$$

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- Near horizon Hamiltonian = boundary charge associated with unit time-translations*

$$H = Q[\partial_t] = \kappa\, P_0$$

  commutes with all generators $\mathcal{J}^\pm_n$

* units defined by specifying κ

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Call it "soft Heisenberg hair"

New entropy formula

Express entropy in terms of near horizon charges:

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- Obeys simple near horizon first law

$$\delta S = \frac{2\pi}{\kappa}\,\delta\big(\kappa P_0\big) \qquad \Rightarrow \qquad T\,\delta S = \delta H$$

with Hawking–Unruh-temperature

$$T = \frac{\kappa}{2\pi}$$

δ refers to any variation of phase space variables allowed by the boundary conditions

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Can we understand entropy law microscopically?

Semi-classical microstates?

Given our soft Heisenberg hair, attack now entropy questions

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3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Regarding 1. and 3.: may expect decoupling of scales so that description of microstates does not need info about UV completion, but rather only some semi-classical "Bohr-like" input

Evidence for this: universality of BH entropy for large black holes

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- possible resolution: do not consider asymptotic but near horizon observer (i.e., employ near horizon bc's and symmetry algebra)

Fluff proposal (with Afshar, Sheikh-Jabbari '16 and also with Yavartanoo '17)

Semi-classical BTZ black hole microstates as near horizon descendants of vacuum

Highest weight vacuum $|0\rangle$

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derived from Bohr-type quantization conditions

- quantization of central charge $c = 3/(2G)$ in integers
- quantization of conical deficit angles in integers over c
- black hole/particle correspondence
  (black hole = gas of coherent states of particles on AdS_3)

Check of fluff proposal

Microstates for BTZ black hole with mass M and angular momentum J:

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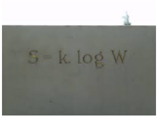

(we set $k=1$ and $W=p$)

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$$S = \ln p\Big(\tfrac{c}{2}\,(M+J)\Big) + \ln p\Big(\tfrac{c}{2}\,(M-J)\Big)$$

- leading order yields Cardy formula and hence the BH entropy

$$S = 2\pi\sqrt{\tfrac{c}{6}\,(M+J)} + 2\pi\sqrt{\tfrac{c}{6}\,(M-J)} = 2\pi\,P_0 = \frac{A}{4G} + \dots$$

Check of fluff proposal

Microstates for BTZ black hole with mass M and angular momentum J:

$$|\mathcal{B}(\{n_i^\pm\})\rangle = \prod_{\{n_i^\pm > 0\}} \left(\mathcal{J}^+_{-n_i^+} \cdot \mathcal{J}^-_{-n_i^-}\right)|0\rangle \qquad \sum_i n_i^\pm = \tfrac{c}{2}\,(M \pm J)$$

- count number of BTZ black hole microstates
- combinatorial problem: how many ways to decompose large positive integer $\frac{c}{2}\,(M \pm J)$ into sum of positive integers
- solved by Hardy, Ramanujan 1918

$$p(N)\big|_{N\gg 1} \sim \frac{1}{4N\sqrt{3}}\,\exp\big(2\pi\,\sqrt{N/6}\big)$$

- to get entropy use Boltzmann's formula

$$S = \ln p\big(\tfrac{c}{2}\,(M+J)\big) + \ln p\big(\tfrac{c}{2}\,(M-J)\big)$$

- leading + subleading order yields BH entropy plus log corrections

$$S = \frac{A}{4G} - 2\ln\big(A/(4G)\big) + \dots$$

Outline

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  works in any dimension, for any local geometry, for any reasonable theory* (with metric) and for any type of non-extremal horizon

* theories checked so far:
Einstein gravity with negative cosmological constant ($d \geq 3$)
Einstein gravity with vanishing cosmological constant ($d \geq 3$)
higher spin gravity ($d = 3$, principal embedding of $sl(2)$)
various massive gravity theories ($d = 3$)

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* for instance, for Schwarzschild

$$\{Q_{lm},\, P_{l'm'}\} = \frac{1}{8\pi G}\,\delta_{ll'}\,\delta_{mm'} \qquad l > 0 \qquad\qquad \{P_{00},\, \bullet\} = 0$$

Q_{lm}: spherical harmonics of area preserving shear deformations
P_{lm}: spherical harmonics of near horizon supertranslations
Entropy given by $S = 2\pi\, P_{00}$

Kerr has additional generators: area preserving twist deformations

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  might work more generally, but so far only checked BTZ black hole; needed Bohr-type rules to succeed

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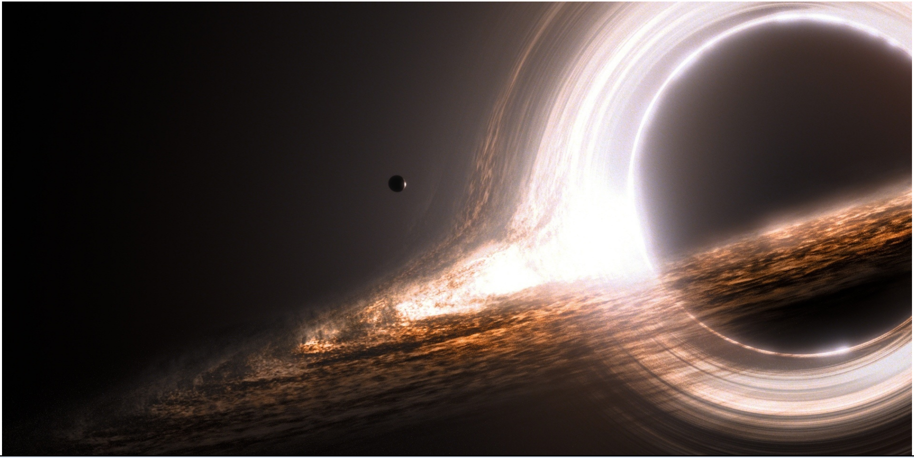
- Microstate construction for non-extremal Kerr?

Thanks for your attention!



Bonus slide I

State-dependence of near horizon Killing vectors

- CS formulation: gauge-parameters $\epsilon^{\pm} = \eta^{\pm}(\varphi)\, L_0$ state-independent

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- thus, Lie-bracket replaced by modified Lie-bracket

$$[\xi_1,\, \xi_2]_{\text{mod}} = [\xi_1,\, \xi_2]_{\text{Lie}} + \delta_{\xi_2}\xi_1 - \delta_{\xi_1}\xi_2$$

main difference to DGGP, where ξ is state-independent!

Bonus slide II

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d} + \hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

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- Virasoro w. Brown–Henneaux central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

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$$\mathcal{J}_{cn} \sim \mathcal{W}_n^0$$

   Note non-local relation

$$\mathcal{W} \sim e^{-2\int \mathcal{J}}$$

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Note twisted periodicity conditions

$$\mathcal{W}^\nu(\varphi + 2\pi) = e^{-2\pi\nu i}\,\mathcal{W}^\nu(\varphi)$$

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Maldacena, Maoz '00; Lunin, Maldacena, Maoz '02

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   Identify states in Hilbert space $\mathcal{H}_{\mathrm{BTZ}}$ as (composite) states in $\mathcal{H}_{\mathrm{CG}}$

$$\sum_p \mathcal{J}_{nc-p}\mathcal{J}_p \sim \sum_p J_{n-p}J_p + incJ_n$$

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   Justification 1: obtain Virasoro at central charge c in $\mathcal{H}_{\text{BTZ}}$ and $\mathcal{H}_{\text{CG}}$
   Justification 2: gives nice result