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CONSISTENT BOUNDARY CONDITIONS FOR COSMOLOGICAL TOPOLOGICALLY MASSIVE GRAVITY AT THE CHIRAL POINT

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We show that cosmological topologically massive gravity at the chiral point allows not only Brown–Henneaux boundary conditions as consistent boundary conditions, but also slightly more general ones which encompass the logarithmic primary found in *J. High Energy Phys.* **07** (2008) 134 as well as all its descendants.

Keywords: Cosmological topologically massive gravity; Brown–Henneaux boundary conditions; chiral gravity; gravity in three dimensions; logarithmic CFT; AdS/CFT.

1. Introduction

Cosmological topologically massive gravity[1] (CTMG) is a three-dimensional theory of gravity that exhibits gravitons[2,3] and black holes.[4] With the sign convention of Ref. 5 its action is given by

$$I_{\mathrm{CTMG}} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\bigg[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\epsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma} \times\left(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\right)\bigg], \tag{1}$$

where the negative cosmological constant is parametrized by $\Lambda = -1/\ell^2$. If the constants μ and ℓ satisfy the condition

$$\mu\ell = 1, \tag{2}$$

the theory is called "CTMG at the chiral point." The condition (2) is special, because one of the central charges of the dual boundary CFT vanishes: $c_L = 0$, $c_R \neq 0$.

This observation was the motivation for Ref. 6 to consider CTMG at the chiral point in some detail. In that work the theory (1) with (2) was dubbed "chiral gravity," assuming that all solutions obey the Brown–Henneaux boundary conditions.[7] Moreover, it was conjectured that CTMG at the chiral point is a chiral theory and that the local physical degree of freedom, i.e. the topologically massive graviton, disappears. These statements were disputed in Ref. 8, which engendered a lot of recent activity concerning CTMG.[5,9–20] In particular, the present authors constructed explicitly[5] a physical mode not considered in Ref. 6 using their formalism. This mode, which we call the "logarithmic primary," violates the Brown–Henneaux boundary conditions. These results were confirmed very recently in Ref. 20, where one of the descendants of the logarithmic primary was considered. It was found that this descendant (and all successive descendants) can be made consistent with the Brown–Henneaux boundary conditions by a diffeomorphism. Thus, these modes are present in classical CTMG (in addition to the standard boundary gravitons), even if Brown–Henneaux boundary conditions are imposed. The latest development is a simple classical proof[21] of the chirality of the generators of diffeomorphisms at $\mu\ell = 1$, concurring with previous results.[19]

In the conclusions of Ref. 21 it was speculated that perhaps there are consistent boundary conditions other than the ones by Brown and Henneaux for CTMG at the chiral point. It is the purpose of this note to show that this is indeed the case and that the new set of boundary conditions encompasses the logarithmic primary.

2. Beyond Brown–Henneaux

We follow as closely as possible the notation and the logical flow of Ref. 21. Any metric consistent with the boundary conditions to be imposed below must be asymptotic to AdS$_3$, which in Poincaré coordinates is given by

$$g^{\mathrm{AdS}}_{\mu\nu}\, dx^\mu dx^\nu = \ell^2 \left(\frac{dx^+ dx^- + dy^2}{y^2} \right), \tag{3}$$

where the boundary is located at $y = 0$. The Brown–Henneaux boundary conditions then require that fluctuations $h_{\mu\nu}$ of the metric about (3) fall off as

$$\begin{pmatrix} h_{++} = \mathcal{O}(1) & h_{+-} = \mathcal{O}(1) & h_{+y} = \mathcal{O}(y) \\ & h_{--} = \mathcal{O}(1) & h_{-y} = \mathcal{O}(y) \\ & & h_{yy} = \mathcal{O}(1) \end{pmatrix}. \tag{4}$$

By $\mathcal{O}(x)$ we mean that the corresponding fluctuation metric component behaves *at most* proportionally to x in the small y limit.

We now define suitable boundary conditions that encompass the logarithmic primary and its descendants. Let us first recall the form of the logarithmic primary and see how the Brown–Henneaux boundary conditions need to be weakened. Translating the result (3.3) of Ref. 5 into Poincaré coordinates yields schematically[a]

$$h^{\text{new}}_{\mu\nu}dx^\mu dx^\nu \sim \mathcal{O}(\log y)(dx^-)^2 + \mathcal{O}(y\log y)dx^- dy + \mathcal{O}(y^2\log y)dy^2. \tag{5}$$

Evidently the logarithmic primary behaves as follows:

$$h^{\text{new}}_{--} = \mathcal{O}(\log y), \quad h^{\text{new}}_{-y} = \mathcal{O}(y\log y), \quad h^{\text{new}}_{yy} = \mathcal{O}(y^2\log y). \tag{6}$$

From (4) we see that the Brown–Henneaux boundary conditions for these three components are

$$h_{--} = \mathcal{O}(1), \quad h_{-y} = \mathcal{O}(y), \quad h_{yy} = \mathcal{O}(1). \tag{7}$$

It is clear that (6) is incompatible with (7). However, only the first two conditions of (3) have to be weakened logarithmically to encompass the logarithmic primary.

Therefore, we propose the following set of boundary conditions[b]:

$$\begin{pmatrix} h^{\text{new}}_{++} = \mathcal{O}(1) & h^{\text{new}}_{+-} = \mathcal{O}(1) & h^{\text{new}}_{+y} = \mathcal{O}(y) \\ & h^{\text{new}}_{--} = \mathcal{O}(\log y) & h^{\text{new}}_{-y} = \mathcal{O}(y\log y) \\ & & h^{\text{new}}_{yy} = \mathcal{O}(1) \end{pmatrix}. \tag{8}$$

Let us determine the diffeomorphisms

$$g_{\mu\nu} = g^{\text{AdS}}_{\mu\nu} + h^{\text{new}}_{\mu\nu} \quad \to \quad \mathcal{L}_\zeta\, g_{\mu\nu} = \tilde{g}_{\mu\nu} = g^{\text{AdS}}_{\mu\nu} + \tilde{h}^{\text{new}}_{\mu\nu}, \tag{9}$$

which preserve these boundary conditions, i.e. we require that $\tilde{h}^{\text{new}}_{\mu\nu}$ should also have the fall-off behavior postulated in (8). Calculating the generator ζ^μ with this requirement yields

$$\zeta^+ = \epsilon^+(x^+) - \frac{y^2}{2}\partial_-^2\epsilon^- + \mathcal{O}(y^4\log y), \tag{10}$$

$$\zeta^- = \epsilon^-(x^-) - \frac{y}{2}\partial_+^2\epsilon^+ + \mathcal{O}(y^4), \tag{11}$$

$$\zeta^y = \frac{y^2}{2}\left(\partial_+\epsilon^+(x^+) + \partial_-\epsilon^-(x^-)\right) + \mathcal{O}(y^3). \tag{12}$$

Remarkably, the only difference to the Brown–Henneaux case is the possibility of an $\mathcal{O}(y^4\log y)$ behavior for the sub-subleading term in the ζ^+ component as opposed to $\mathcal{O}(y^4)$; see e.g. (5)–(8) in Ref. 21. Thus, there are transformations that

[a]The coordinates in that work are related to the coordinates here as follows: $x^\pm = (\phi \mp \tau)/2$, $y \sim e^{-\rho}$.

[b]The proposal (8) may be compared with footnote 3 of Ref. 21: it is not necessary to weaken the boundary conditions of all components $h_{\pm\pm}$ to $\mathcal{O}(\ln y)$ (see first sentence) and it is not sufficient to take only h_{--} to be $\mathcal{O}(\ln y)$ (see second sentence).

preserve the new set of boundary conditions (8) but not the Brown–Henneaux set of boundary conditions (4). These new transformations must still be considered pure gauge because of their rapid fall-off near the boundary.

Thus, we end up with the following situation. The suitable boundary conditions for encompassing the logarithmic primary are given by (8) rather than by (4). These are preserved by more gauge transformations than the Brown–Henneaux conditions, but exhibit the same asymptotic symmetries. Since the isometry algebra of AdS_3 is part of the transformations that preserve (8), and since the descendants are produced by acting with this algebra, we automatically demonstrate that all descendants of h^{new} fulfill (8).

3. Boundary Stress Tensor and Asymptotic Symmetry Generators

It is also important that all metrics fulfilling (8) have well-defined generators of the asymptotic symmetries. This can be shown as follows. We compute the boundary stress tensor along the lines of Ref. 5 and find that it reduces to the Kraus–Larsen result[22]:

$$T_{++} = \frac{1}{4\pi G\,\ell}\,h^{\text{new}}_{++} \sim \mathcal{O}(1), \tag{13}$$

$$T_{--} = 0, \tag{14}$$

$$T_{+-} = 0. \tag{15}$$

Note that the result above coincides with (10)–(12) of Ref. 21 for $\mu\ell = 1$. The off-diagonal contribution T_{+-} vanishes after one imposes constraints from the equations of motion. The generators of the asymptotic symmetry group become

$$Q[\zeta] = \frac{1}{4\pi G\,\ell}\int dx^+ h^{\text{new}}_{++}\epsilon^+ \sim \mathcal{O}(1). \tag{16}$$

Since no divergences arise, the generators (16) are well defined.

Thus, we conclude that there are indeed consistent boundary conditions (8) that go beyond Brown–Henneaux and that allow for the logarithmic primary and all its descendants. Because of the analysis in Sec. 4 of Ref. 5 this result might have been anticipated: there it was shown that the logarithmic primary is consistent with the requirement of space–time being asymptotically AdS and that the ensuing boundary stress tensor is finite, traceless and chiral.

We close by noting that there are other examples where the Brown–Henneaux boundary conditions need to be weakened logarithmically to encompass physically interesting solutions.[23] The boundary conditions of Ref. 23 are not identical to the ones considered in the present note.

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References

1. S. Deser, Cosmological topological supergravity. PRINT-82-0692 (Brandeis).
2. S. Deser, R. Jackiw and S. Templeton, Three-dimensional massive gauge theories, *Phys. Rev. Lett.* **48** (1982) 975.
3. S. Deser, R. Jackiw and S. Templeton, Topologically massive gauge theories, *Ann. Phys.* **140** (1982) 372 [erratum: *ibid.* **185** (1988) 406, APNYA, 281, 409 (1988), APNYA, 281, 409–449 (2000)].
4. M. Banados, C. Teitelboim and J. Zanelli, The black hole in three-dimensional space–time, *Phys. Rev. Lett.* **69** (1992) 1849 [arXiv:hep-th/9204099].
5. D. Grumiller and N. Johansson, Instability in cosmological topologically massive gravity at the chiral point, *J. High Energy Phys.* **0807** (2008) 134 [arXiv:0805.2610 (hep-th)].
6. W. Li, W. Song and A. Strominger, Chiral gravity in three dimensions, *J. High Energy Phys.* **0804** (2008) 082 [arXiv:0801.4566 (hep-th)].
7. J. D. Brown and M. Henneaux, Central charges in the canonical realization of asymptotic symmetries: An example from three-dimensional gravity, *Commun. Math. Phys.* **104** (1986) 207.
8. S. Carlip *et al.*, Cosmological topologically massive gravitons and photons [arXiv:0803.3998 (hep-th)].
9. K. Hotta *et al.*, Brown–Henneaux's canonical approach to topologically massive gravity, *J. High Energy Phys.* **0807** (2008) 066 [arXiv:0805.2005 (hep-th)].
10. W. Li, W. Song and A. Strominger, Comment on "Cosmological Topological Massive Gravitons and Photons" [arXiv:0805.3101 (hep-th)].
11. M. I. Park, Constraint dynamics and gravitons in three dimensions, *J. High Energy Phys.* **09** (2008) 084 [arXiv:0805.4328 (hep-th)].
12. I. Sachs and S. N. Solodukhin, Quasi-normal modes in topologically massive gravity, *J. High Energy Phys.* **0808** (2008) 003 [arXiv:0806.1788 (hep-th)].
13. D. A. Lowe and S. Roy, Chiral geometries of $(2+1)$-d AdS gravity, *Phys. Lett. B* **668** (2008) 159 [arXiv:0806.3070 (hep-th)].
14. D. Grumiller, R. Jackiw and N. Johansson, Canonical analysis of cosmological topologically massive gravity at the chiral point [arXiv:0806.4185 (hep-th)].
15. S. Carlip *et al.*, Topologically massive AdS gravity, *Phys. Lett. B* **666** (2008) 272 [arXiv:0807.0486 (hep-th)].
16. I. Sachs, Quasi-normal modes for logarithmic conformal field theory, *J. High Energy Phys.* **09** (2008) 073 [arXiv:0807.1844 (hep-th)].
17. G. W. Gibbons, C. N. Pope and E. Sezgin, The general supersymmetric solution of topologically massive supergravity, *Class. Quant. Grav.* **25** (2008) 205005 [arXiv:0807.2613 (hep-th)].

18. D. Anninos *et al.*, Warped AdS_3 black holes [arXiv:0807.3040 (hep-th)].
19. S. Carlip, The constraint algebra of topologically massive AdS gravity, *J. High Energy Phys.* **10** (2008) 078 [arXiv:0807.4152 (hep-th)].
20. G. Giribet, M. Kleban and M. Porrati, Topologically massive gravity at the chiral point is not unitary, *J. High Energy Phys.* **10** (2008) 045 [arXiv:0807.4703 (hep-th)].
21. A. Strominger, A simple proof of the chiral gravity conjecture [arXiv:0808.0506 (hep-th)].
22. P. Kraus and F. Larsen, Holographic gravitational anomalies, *J. High Energy Phys.* **0601** (2006) 022 [arXiv:hep-th/0508218].
23. M. Henneaux *et al.*, Asymptotically anti-de Sitter spacetimes and scalar fields with a logarithmic branch, *Phys. Rev. D* **70** (2004) 044034 [arXiv:hep-th/0404236].