Flat Space Holography

Daniel Grumiller

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Gottfried Wilhelm Leibniz Universität Hannover, December 2015

based on work w. Afshar, Bagchi, Basu, Detournay, Fareghbal, Gary, Merbis, Riegler, Rosseel, Salzer, Sarkar, Schöller, Simon, ...

Some of our papers on flat space holography

- A. Bagchi, D. Grumiller and W. Merbis, "Stress tensor correlators in three-dimensional gravity," arXiv:1507.05620. [see arXiv today]
- A. Bagchi, R. Basu, D. Grumiller and M. Riegler, "Entanglement entropy in Galilean conformal field theories and flat holography," Phys. Rev. Lett. 114 (2015) 11, 111602 [arXiv:1410.4089].
- H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel, "Spin-3 Gravity in Three-Dimensional Flat Space," Phys. Rev. Lett. 111 (2013) 12, 121603 [arXiv:1307.4768].



- 🔈 A. Bagchi, S. Detournay, D. Grumiller and J. Simon, "Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space," Phys. Rev. Lett. 111 (2013) 18, 181301 [arXiv:1305.2919].
- 🚬 A. Bagchi, S. Detournay and D. Grumiller, "Flat-Space Chiral Gravity," Phys. Rev. Lett. 109 (2012) 151301 [arXiv:1208.1658].

Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues

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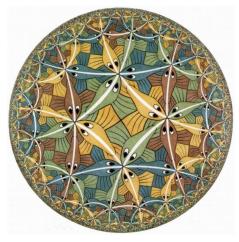
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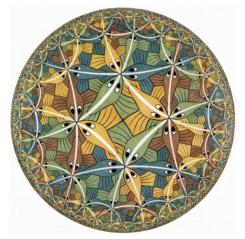
Holography beyond AdS/CFT?

This talk focuses on holography.



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Main question: how general is holography?

How general is holography?

- Holographic principle realized in AdS/CFT correspondence
- Special case or generic lesson for quantum gravity?

$\mathsf{AdS}_{d+1} \to \mathsf{CFT}_d$

- Use (classical) gravity to learn more about CFTs
- Strong coupling large N limit: classical gravity
- Useful tool to calculate correlation functions
- Useful tool to calculate entanglement entropy

 $\mathsf{CFT}_d \to \mathsf{AdS}_{d+1}$

- Use CFTs to learn more about (quantum) gravity
- Gravity in ultra-quantum limit: simple CFT?
- Useful tool to address black hole microstates
- Useful tool for qu-gr puzzles (information paradox)

How general is holography?

Historical curiosity: Karl Schwarzschild (1873-1916)



Simplest of all (classical) black holes: Schwarzschild solution (22.12.1915)

$$ds^{2} = (1 - \gamma/R) dt^{2} - \frac{dR^{2}}{1 - \gamma/R}$$
$$- R^{2} (d\theta^{2} + \sin^{2}\theta d\phi^{2})$$

(Schwarzschild's conventions in letter to Einstein)

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- Quantum mechanically: among least understood black holes!
- Microstates?
- Log corrections to entropy?
- Dual field theory??

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How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop "Bits, Branes, Black Holes" 2012

and at ESI workshop "Higher Spin Gravity" 2012

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- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson '08; Skenderis, Taylor, van Rees '09; Henneaux, Martinez, Troncoso '09; Maloney, Song, Strominger '09; DG, Sachs/Hohm '09; Gaberdiel, DG, Vassilevich '10; ... DG, Riedler, Rosseel, Zojer '13
- recent proposal by Vafa '14

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- Can we establish a flat space holographic dictionary?

the answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., '12-'15

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- Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work at least in 2+1 dimensions Gary, DG Rashkov '12; Afshar et al '12; Gutperle et al '14-'15; Gary, DG, Prohazka, Rey '14; ...

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 - Address questions above in simple class of 3D toy models
 - Exploit gauge theoretic Chern–Simons formulation
 - Restrict to kinematic questions, like (asymptotic) symmetries

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Address these issues in 3D!



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Interesting dichotomy:

- \blacktriangleright Either dual field theory exists \rightarrow useful toy model for quantum gravity
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This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding

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- Simple microstate counting from AdS₃/CFT₂

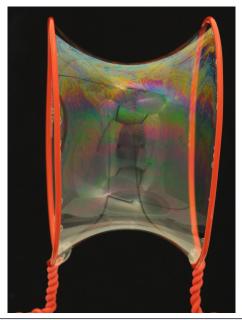
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Caveat: while there are many string compactifications with AdS_3 factor, applying holography just to AdS_3 factor does not capture everything!

Picturesque analogy: soap films



Both soap films and Chern–Simons theories have

- essentially no bulk dynamics
- highly non-trivial boundary dynamics
- most of the physics determined by boundary conditions
- esthetic appeal (at least for me)



Daniel Grumiller — Flat Space Holography

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- Take linear combinations of Virasoro generators \mathcal{L}_n , $\overline{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}$$
 $M_n = \frac{1}{\ell} \left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n} \right)$

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• Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$
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This is nothing but the BMS₃ algebra (or GCA₂, URCA₂, CCA₂)! Ashtekar, Bicak, Schmidt, '96; Barnich, Compere '06 L_n: diffeos of circle, M_n: supertranslations, c_{L/M}: central extensions

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 This is nothing but the BMS₃ algebra (or GCA₂, URCA₂, CCA₂)! If dual field theory exists it must be a 2D Galilean CFT! Bagchi et al., Barnich et al.

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- Example where it does not work: highest weight conditions

See also talk by Mirah Gary two days ago (Junior plenary session)

▶ AdS gravity in CS formulation: $sl(2) \oplus sl(2)$ gauge algebra

Achucarro, Townsend '86; Witten '88

- ▶ AdS gravity in CS formulation: $sl(2) \oplus sl(2)$ gauge algebra
- ► Flat space: isl(2) gauge algebra

$$S_{\rm CS}^{\rm flat} = rac{k}{4\pi} \int \langle \mathcal{A} \wedge \mathrm{d}\mathcal{A} + rac{2}{3} \,\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}
angle$$

with isl(2) connection $(a = 0, \pm 1)$

$$\mathcal{A} = e^a M_a + \omega^a L_a$$

 $\mathsf{isl}(2)$ algebra (global part of $\mathsf{BMS}/\mathsf{GCA}$)

$$[L_a, L_b] = (a - b)L_{a+b}$$
$$[L_a, M_b] = (a - b)M_{a+b}$$
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Note: e^a dreibein, ω^a (dualized) spin-connection Bulk EOM: gauge flatness \rightarrow Einstein equations

$$\mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = 0$$

- ▶ AdS gravity in CS formulation: $sl(2) \oplus sl(2)$ gauge algebra
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Boundary conditions in CS formulation:

$$\mathcal{A}(r, u, \varphi) = b^{-1}(r) \left(d + a(u, \varphi) + o(1) \right) b(r)$$

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► Flat space boundary conditions: $b(r) = \exp\left(\frac{1}{2}rM_{-1}\right)$ and $a(u, \varphi) = \left(M_1 - M(\varphi)M_{-1}\right) du + \left(L_1 - M(\varphi)L_{-1} - N(u, \varphi)M_{-1}\right) d\varphi$ with $N(u, \varphi) = L(\varphi) + \frac{u}{2}M'(\varphi)$

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 Flat space boundary conditions: b(r) = exp (¹/₂ rM₋₁) and a(u, φ) = (M₁ − M(φ)M₋₁) du + (L₁ − M(φ)L₋₁ − N(u, φ)M₋₁) dφ with N(u, φ) = L(φ) + ^u/₂ M'(φ)
 metric

$$g_{\mu\nu} \sim \frac{1}{2} \,\widetilde{\mathrm{tr}} \langle \mathcal{A}_{\mu} \mathcal{A}_{\nu} \rangle \quad \rightarrow \quad \mathrm{d}s^2 = M \,\mathrm{d}u^2 - 2 \,\mathrm{d}u \,\mathrm{d}r + 2N \,\mathrm{d}u \,\mathrm{d}\varphi + r^2 \,\mathrm{d}\varphi^2$$

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What is flat space analogue of

$$\langle T(z_1)T(z_2)\ldots T(z_{42})\rangle_{\rm CFT} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\rm EH-AdS}\Big|_{\rm EOM}$$

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 $\label{eq:AdS/CFT} \begin{array}{l} \mbox{good tool for calculating correlators} \\ \mbox{What about flat space/Galilean CFT correspondence?} \end{array}$

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Start slowly with 0-point function

Not check of flat space holography but interesting in its own right

 \blacktriangleright Calculate the full on-shell action Γ

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- \blacktriangleright Calculate the full on-shell action Γ
- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N} \int \mathrm{d}^3 x \sqrt{g} \, R - -\frac{1}{8\pi G_N} \int \mathrm{d}^2 x \sqrt{\gamma} \, K - I_{\text{counter-term}}$$

with $I_{\rm counter-term}$ chosen such that

$$\delta \Gamma \big|_{\rm EOM} = 0$$

for all δg that preserve flat space bc's

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with $I_{\mathrm{counter-term}}$ chosen such that

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for all δg that preserve flat space bc's Result (Detournay, DG, Schöller, Simon '14):

$$\Gamma = -\frac{1}{16\pi G_N} \int \mathrm{d}^3 x \sqrt{g} \, R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\mathrm{GHY!}} \int \mathrm{d}^2 x \sqrt{\gamma} \, K$$

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli '04 independently confirmed by Barnich, Gonzalez, Maloney, Oblak '15

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Not check of flat space holography but interesting in its own right

- \blacktriangleright Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?

Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

Evaluate Euclidean partition function in semi-classical limit

$$Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}}$$

path integral bc's specified by temperature T and angular velocity $\boldsymbol{\Omega}$

Two Euclidean saddle points in same ensemble if

- \blacktriangleright same temperature $T=1/\beta$ and angular velocity Ω
- obey flat space boundary conditions
- solutions without conical singularities

Periodicities fixed:

$$(\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta\Omega) \sim (\tau_E, \varphi + 2\pi)$$

Not check of flat space holography but interesting in its own right

- \blacktriangleright Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?
 3D Euclidean Einstein gravity: for each T, Ω two saddle points:
 - Hot flat space

$$\mathrm{d}s^2 = \mathrm{d}\tau_E^2 + \mathrm{d}r^2 + r^2 \,\mathrm{d}\varphi^2$$

Flat space cosmology

$$\mathrm{d}s^{2} = r_{+}^{2} \left(1 - \frac{r_{0}^{2}}{r^{2}}\right) \,\mathrm{d}\tau_{E}^{2} + \frac{r^{2} \,\mathrm{d}r^{2}}{r_{+}^{2} \left(r^{2} - r_{0}^{2}\right)} + r^{2} \left(\mathrm{d}\varphi - \frac{r_{+}r_{0}}{r^{2}} \,\mathrm{d}\tau_{E}\right)^{2}$$

shifted-boost orbifold, see Cornalba, Costa '02

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- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\rm HFS} = -\frac{1}{8G_N} \qquad F_{\rm FSC} = -\frac{r_+}{8G_N}$$

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- Result of this comparison
 - $r_+ > 1$: FSC dominant saddle
 - $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS "melts" into FSC at $T > T_{c} \label{eq:FSC}$

Bagchi, Detournay, DG, Simon '13

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3 :

$$\delta \Gamma \big|_{\rm EOM} \sim \int_{\partial \mathcal{M}} {\rm vev} \times \delta \, {\rm source} \sim \int_{\partial \mathcal{M}} T^{\mu\nu}_{\rm BY} \times \delta g^{\rm NN}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\rm BY}$ follows from canonical analysis as well (conserved charges)

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- analogue of Brown–York stress tensor?
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everything works (Detournay, DG, Schöller, Simon, '14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G} \qquad N = \frac{g_{t\varphi}}{4G}$$

full tower of canonical charges: see Barnich, Compere '06

2-point functions (anomalous terms) First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder
$$(\varphi \sim \varphi + 2\pi)$$
:
 $\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0$
 $\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}$
 $\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Fourier modes of Galilean CFT stress tensor on cylinder:

$$M := \sum_{n} M_{n} e^{-in\varphi} - \frac{c_{M}}{24}$$
$$N := \sum_{n} (L_{n} - inuM_{n})e^{-in\varphi} - \frac{c_{L}}{24}$$

Conservation equations: $\partial_u M = 0$, $\partial_u N = \partial_{\varphi} M$

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Summarize first how this works in the AdS case

Illustrate shortcut in AdS_3/CFT_2 (restrict to one holomorphic sector)

• On CFT side deform free action S_0 by source term μ for stress tensor

$$S_{\mu} = S_0 + \int \mathrm{d}^2 z \,\mu(z,\bar{z})T(z)$$

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$$A = b^{-1}(\mathbf{d}+a)b \qquad b = e^{\rho L_0}$$
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Drinfeld, Sokolov '84, Polyakov '87, H. Verlinde '90 Bañados, Caro '04

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$$\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$$

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▶ Solve them using the Green function on the plane $G = \ln (z_{12}\bar{z}_{12})$

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- Generalize to cylinder

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Solve with Green function on cylinder

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• Correct 2-point functions for Einstein gravity with $c_L = 0$, $c_M = 12k$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

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> Yes: same procedure, but localize chemical potentials at two points

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Iteratively solve EOM

$$\begin{aligned} \partial_u M &= -k\partial_{\varphi}^3 \mu_L + \mu_L \partial_{\varphi} M + 2M \partial_{\varphi} \mu_L \\ \partial_u N &= -k\partial_{\varphi}^3 \mu_M + (1+\mu_M) \partial_{\varphi} M + 2M \partial_{\varphi} \mu_M + \mu_L \partial_{\varphi} N + 2N \partial_{\varphi} \mu_L \end{aligned}$$

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Result on gravity side matches precisely Galilean CFT results

$$\langle M^1 \, N^2 \, N^3 \rangle = \frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2} \qquad \langle N^1 \, N^2 \, N^3 \rangle = \frac{c_L - c_M \, \tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2}$$

provided we choose again the Einstein values $c_L = 0$ and $c_M = 12k$

4-point functions (enter cross-ratios) First correlators with non-universal function of cross-ratios

Repeat this algorithm, localizing the sources at three points

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- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1 N^2 N^3 N^4 \rangle = \frac{2c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}^2} \\ \langle N^1 N^2 N^3 N^4 \rangle = \frac{2c_L g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}^2}$$

with the cross-ratio function

$$g_4(\gamma) = rac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = rac{s_{12} \, s_{34}}{s_{13} \, s_{24}}$$

and

$$\Delta_4 = 4g'_4(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)$$

$$\eta_{1234} = \sum (-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)$$

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Repeat this algorithm, localizing the sources at four points

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- Derive 5-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

$$\langle M^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_M g_5(\gamma, \zeta)}{\prod_{1 \le i < j \le 5} s_{ij}} \langle N^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_L g_5(\gamma, \zeta) + c_M \Delta_5}{\prod_{1 \le i < j \le 5} s_{ij}}$$

with the previous definitions and $(\zeta=\frac{s_{25}\,s_{34}}{s_{35}\,s_{24}})$

$$g_{5}(\gamma,\,\zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{(\gamma^{2}-\gamma\zeta+\zeta^{2})}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)} \times \left([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta \right)$$
$$\Delta_{5} = 4\partial_{\gamma}g_{5}(\gamma,\,\zeta)\eta_{1234} + 4\partial_{\zeta}g_{5}(\gamma,\,\zeta)\eta_{2345} - 2g_{5}(\gamma,\,\zeta)\tau_{12345}$$

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with the previous definitions and $\bigl(\zeta=\frac{s_{25}\,s_{34}}{s_{35}\,s_{24}}\bigr)$

$$g_5(\gamma,\,\zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{(\gamma^2-\gamma\zeta+\zeta^2)}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)} \times \left([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta \right)$$

$$\Delta_5 = 4\partial_\gamma g_5(\gamma,\zeta)\eta_{1234} + 4\partial_\zeta g_5(\gamma,\zeta)\eta_{2345} - 2g_5(\gamma,\zeta)\tau_{12345}$$

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Smart check of all *n*-point functions?

▶ Idea: calculate n-point function from (n-1)-point function

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- Need Galilean CFT analogue of BPZ-recursion relation

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We can also derive same recursion relations on gravity side!

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Fairly non-trivial check that 3D flat space holography can work!

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$$S_{\rm EE}^{\rm GCFT} = \underbrace{\frac{c_L}{6} \ln \frac{\ell_x}{a}}_{\rm like \ CFT} + \underbrace{\frac{c_M}{6} \frac{\ell_y}{\ell_x}}_{\rm like \ grav \ anomaly}$$

Calculation on gravity side confirms result above (using Wilson lines in CS formulation)

Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues

Recent generalizations:

adding chemical potentials

Works! (Gary, DG, Riegler, Rosseel '14)

In CS formulation:

$$A_0 \to A_0 + \mu$$

See also talk by Mirah Gary two days ago (Junior plenary session)

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)

Conformal CS gravity at level k = 1 with flat space boundary conditions conjectured to be dual to chiral half of monster CFT. Action (gravity side):

$$I_{\rm CSG} = \frac{k}{4\pi} \int d^3x \sqrt{-g} \, \varepsilon^{\lambda\mu\nu} \, \Gamma^{\rho}{}_{\lambda\sigma} \left(\partial_{\mu} \Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3} \, \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho} \right)$$

Partition function (field theory side, see Witten '07):

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

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- generalization to supergravity

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Asymptotic symmetry algebra = super-BMS $_3$

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- flat space higher spin gravity

Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13)

New type of algebra: W-like BMS ("BMW")

$$[U_n, U_m] = (n-m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m}$$
$$- \frac{96(c_L + \frac{44}{5})}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}$$
$$[U_n, V_m] = (n-m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M}(n-m)\Theta_{n+m}$$
$$+ \frac{c_M}{12}n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}$$
$$, L], [L, M], [M, M] \text{ as in BMS}_3 \qquad [L, U], [L, V], [M, U], [M, V] \text{ as in isl}(3)$$

L

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Some open issues:

▶ Further checks in 3D (*n*-point correlators, partition function, ...)

Barnich, Gonzalez, Maloney, Oblak '15: 1-loop partition function matches BMS_3 character

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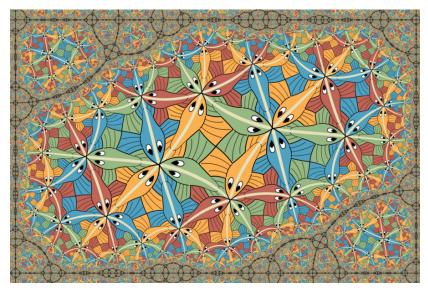
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 - holography seems to work in flat space
 - holography more general than AdS/CFT
 - (when) does it work even more generally?

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle